LEVELING AND EARTHWORK

oseph aker

J. B. DAVIS

1873 TO 1906

ANN ARBOR, MICHIGAN

1906

LEVELS

1. **Center Line** marked by suitable numbered stakes equally spaced, numbers all facing in the direction in which they decrease.

2. **Levels.** After the center line is marked as above noted levels are to be taken on the ground along it, for the purpose of finding the changes in elevation, or slope, of the surface of the ground. This is done so one can tell what alterations of the surface are needed to bring it to any desired inclination, or grade line. Lines may be run and levels taken for other objects besides actual building, or grading. They may be run for the purpose of making comparisons of different lines of communication as to amount of earthwork and character of materials to be moved, together with length, grades, and other important considerations. Here, however, attention is confined to the amount of materials to be moved. Lines may also be run simply to show the impracticability of a route. When lines are run for purposes of comparison or to show impracticability, it is not necessary to take the levels with the same precision as is needed when they are to be the basis of construction, and pay for the work. Lines run to build by are called located lines, and the work of running them, location. Other lines are called preliminary lines. Some run located lines, that is use the precision of location, on all lines in an easy country. This takes but little extra time, probably none in a timbered region, and saves rerunning the line chosen for construction. Time and expense are thus saved. In running lines to show impracticability, little may be required of the levels beyond showing the object sought.

3. **Levels Datum.** Levels run as indicated are called continuous levels. They are referred to a level surface,—not a plane,—usually below the entire work, called a datum. When the levels are completed the notes will show the actual elevation above datum of every place observed on.

4. **Levels, Benches.** The position of the datum with reference to surrounding obects at any designated locality is preserved by carefully observing, and recording the elevation of some secure object from it. Such objects are called bench marks or simply benches. They are designated in the notes, and elsewhere, by the letters B. M. They should be fixed objects, easily identified, and a full descriptive record of them preserved. The watertable of a masonry building, a suitable mark on a driven pile, are examples of desirable benches. Many times such points cannot be obtained, as the leveler may be far from constructions of any kind. In timber an excellent bench can be made on the buttress formed where the root of a large tree joins the trunk. To do this begin by cutting two notches, one on each side of the buttress, and meeting so as to leave a pointed spur sticking up out of the root. Clip off the top of this spur to form a small seat for the level rod. Two objects should be kept in view. Form the bench so the rod **cannot** be set on the wrong place, and so this place cannot soon be covered by the healing of the wound in the tree. All benches should be so taken that thereafter no mistake can be made as to the original place the rod was set on. Over a timberless region it is necessary to carry all stakes used and to make benches by driving a stake into the ground until it stands firm. The top of such a stake should be used, and rounded for this purpose. The bench stake should be driven down within a couple of inches of being covered, and a flat broad stake driven beside it, properly marked, to show its place. Of course such a bench must be described with reference to the center line of the work as this will be the main object of reference.

5. **Levels.** In beginning a new piece of work it is customary to assume a datum so low that it will surely come below the lowest point of the work. Where the lines to be run

extend over long distances aid in doing this can be obtained from maps by noting how the water runs, and where the mountains, lakes and other natural features are, that indicate differences of level. Many times there may be barometrical levels accessible. In assuming a datum it is customary to refer it to some existing natural object. When near the ocean the object very properly chosen is the water at mean tide, as determined by the Coast Survey where this is accessible. In the vicinity of the great lakes they are considered. A small lake—a pond—the top of a rocky ledge at a certain place, or any similar feature may be used. This is done in order that all the levels in a given district, state, or country, may be easily connected and at last referred to the same datum. It also aids in connecting the elevations of the natural features of the country. Having determined the position of the datum in a general way it is customary to assume it at some even number of tens of feet below a designated permanent bench mark of which a detailed descriptive record should be made and carefully preserved. In starting a line from a great height to run to lower ground it may be found necessary to assume a datum some thousands of feet below the starting bench. This would introduce large numbers into the level notes and become inconvenient. It will therefore be better to change the datum, even thousands of feet at well designated points, as the survey proceeds and begin with a datum much higher than would otherwise be necessary. The same device may be used in ascending. When a survey begins at a place where there is a bench whose elevation above datum is already determined, that would be used in starting.

6. **For running levels,** there is required a good level and rod; a hatchet, of the axe pattern, for making pegs and benches; a note book and waterproof sling pocket for both leveler and rodman; hard pencils, rubber erasers, and a supply of adjusting pins. A waterproof cover for the level is very desirable. The wye level is preferable because of the ease of adjustment. The leveler should regularly see, every morning before work begins, that his level is in correct adjustment. Whenever there is any cause to doubt the adjustments they should be examined at once and corrected if they need it.

Such cases may arise when the level has received an accidental blow or shock. The rod used will be a matter of individual choice. The speaking rod will be found much better than the target rod. The practice with the target rod will be given below because it will be found easier to adapt the instructions given to speaking rod practice than it would to make the corresponding change the other way. The leveler and rodman should both keep a set of notes. The levelers should be in full as to everything on his work. The rodman's should be in full as to everything pertaining to the benches, turning points, and heights of instrument at the various settings. Useful forms follow.

CONTINUOUS LEVEL NOTES.

Juniper Hill and Sunny Grove R. R. Line L.

+ S.	H. I.	— S.	Elevation.	Objects.	
			104.632	B. M.	Root of
4.785	109.417				
		3.16	106.26	Sta 0	Line L =
		4.78	104.64	1	
		6.32	103.10	2	
		5.46	103.96	+ 40	
		8.27	101.15	3	
		6.24	103.18	+ 25	
		5.931	103.486	+ 45	T. P.
6.421	109.907				
		6.21	103.70	4	
		7.42	102.39	5	
		10.68	99.23	+ 60	Bend of
		9.31	100.60	6	
		8.24	101.68	7	
		10.432	99.475	B. M.	Root of
		7.28	102.63	8	
		6.38	103.53	9	
		7.84	102.07	+ 40	Center of
		10.365	99.542	O K 10	T. P.
0.861	100.403				
		8.26	92.14	+ 75	Top of

LEVELER'S BOOK.

Location—Maple Bough Township.

1890-7-14.

F. WALKER, Leveler. S. POKE, Rodman.

Level, Plumbob and Linestaff, 2821.

16″ Beech, 65′ left of 748 + 75, Line B.

Sta 745 + 67.3, Line B.

Proof.

+ S	— S
4.785	
6.421	5.931
	10.365
104.632	99.542
115.838	115.838

Placid River (200′ wide) 275′ L.

14″ Hard Maple, 80′ R. of 7 + 70 where line leaves woods.

highway.

bank of small stream running to R.

Juniper Hill and Sunny Grove R. R. Line L.

+ S.	H. I.	— S.	Elevation.	Objects.	
	100.403				
		10.4	90.00	+ 76	Bottom of
		11.3	89.1	+ 78	Center of
		10.6	89.8	+ 81	Bottom of
		8.42	91.98	+ 83	Top of
		7.84	92.56	11	
		8.32	92.08	+ 30	
		11.70	89.233	+ 50	T. P.
0.417	89.650				
		10.978	78.672		T. P.
0.396	79.068				
		9.294	69.774	+ 70	T. P.
		8.437	70.631	B. M.	Root of
		10.496	68.572	+ 95	Edge of
		11.1	68.0	12	Bottom of
Round soft marsh			64.0	13	4.6 Depth
4.633		4.557	60.7	14	7.9 "
3.741		3.817	57.9	15	10.7 "
4.692		4.713	60.5	16	8.1 "
13.066		13.087	62.0	17	6.6 "
	79.047	0.021	66.0	18	2.6 "
		10.477	68.570	+ 48	Edge of
		9.20	69.8	+ 60	Bottom of
		0.368	78.679	OK BM	× on a

Location—Maple Bough Township.

1890-7-14

F. WALKER, Leveler. S. POKE, Rodman.

Level, Plumbob and Linestaff, 2821.

bank.	**Proof.**	
channel. Water 1′.9 deep.	+ S	— S
bank	0.861	
bank.	0.417	11.170
	0.396	10.978
	13.066	13.087
Top of bank of Crystal Lake.		0.368
	99.542	78.679
	114.282	114.282

Bottom of bank.
27″ Elm 75′ L of 11 + 75.
water. Water level.
lake. Sand. Firm.

of water.
" "
" "
" "

" "

" "

water. Water level.
bank.
6 ft. boulder, 2 ft. out of ground, 80′ R. Sta. 19.

+ S.	H. I.	— S.	Elevation.	Objects.	
				78.679	B.M.
9.433	88.112				
		7.83	80.28	19	
		4.69	83.42	+ 35	Top of
		6.657	81.455	B. M.	Root of
		3.73	84.38	+ 75	
		0.896	87.216	20	T. P.
8.472	95.688				
		7.22	88.47	21	In edge
		5.72	89.97	22	
		5.21	90.48	+ 30	
		4.32	91.37	+ 80	
		6.61	89.08	23	
		7.51	88.18	+ 40	
		5.66	90.03	+ 55	
		3.97	91.72	24	
		0.000	95.688	+ 45	T. P. Peg
10.661	106.349				
		6.64	99.71	+ 65	
		4.73	101.62	25	
		3.627	102.722	B. M.	Top of
		3.07	103.28	+ 12	
		0.318	106.031	+ 35	T. P.
11.014	117.045	O. K.			

Location—Maple Bough Township.

1890-7-14.

F. WALKER, Leveler. S. POKE, Rodman.

Level, Plumbob and Linestaff, 2821.

	Proof.	
	+ S	— S
bank, Crystal Lake.	9.433	
10″ Red Oak, 75′ L. 19 + 50.	8.472	0.896
	10.661	0.000
	11.014	0.318
	78.679	117.045
of corn field.		
	118.259	118.259

driven to elevation of cross wire.

W. T. stone fdn. S. E. cor. brick building, 200′ R. Sta. 25.

+ S.	H. I.	— S.	Elevation.	Objects.	
	117.045				
		7.62	109.42	+ 55	
		4.71	112.34	26	
		1.42	115.63	+ 55	
		0.000	117.045	Peg.	T. P.
11.026	128.071				
		10.79	117.28	27	
		10.07	118.00	+ 15	
		7.10	120.97	+ 40	
		3.60	124.47	+ 60	
		1.80	126.27	+ 65	
		1.01	127.06	28	
		0.000	128.071	Peg.	T. P.
11.262	139.333				
		10.47	128.86	+ 25	
		8.66	130.67	29	
		3.71	135.62	+ 90	
		3.61	135.72	30	
		3.02	136.31	+ 33	
		1.83	137.50	+ 50	
		1.80	137.53	+ 70	
		2.11	137.22	31	
		2.15	137.18	+ 55	
		1.978	137.355	B. M.	Top cf

O. K.

Location—Maple Bough Township.

1890-7-14.

F. WALKER, Leveler. S. POKE, Rodman.

Level, Plumbob and Linestaff, 2821.

	Proof.	
	+ S	— S
	11.026	0.000
Driven to elevation of cross wire.	11.262	0.000
		1.978
	117.045	137.355
	139.333	139.333

Driven to elevation of cross wire.

Summit.

cap stone (S. W. Cor.), bridge seat, E. end highway bridge, over Roaring Creek, about 800 ft. W. of J. **Smith's house.**

Juniper Hill and Sunny Grove R. R. Line L.

+ S.	H. I.	— S.	Elevation.	Objects.	
O. K.	139.333				
		3.25	136.08	32	
		5.14	134.14	+ 70	
		4.96	134.37	33	
		4.02	135.31	+ 62	
		5.23	134.10	34	
		7.51	131.82	+ 60	
Boston Rod.		11.17	128.16	+ 80	
+6.100		10.307	129.026		T. P.
—6.627		11.40	127.93	35	
—0.527	128.499				
		2.23	126.27	+ 70	
		4.64	123.86	+ 75	
		6.23	122.27	36	
		8.16	120.34	+ 50	
		9.63	118.87	37	
		9.281	119.218	B. M.	Root of
0.371	119.589				
		11.62	117.97	+ 18	
		10.755	108.834	+ 70	T. P.
0.363	109.197	O. K.			
		1.08	108.12	38	
		4.12	105.08	39	
		4.17	105.03	+ 45	

Location—Maple Bough Township.

1890-7-14.

F. WALKER, Leveler. S. POKE, Rodman.

Level, Plumbob and Linestaff, 2821.

Transit ⊙ in fence at N. E. side of cornfield. Enter brush pasture.

	Proof.	
Top of stake at Sta. 35.	+ S	— S
	139.333	10.307
		0.527
	0.371	9.281
	0.363	10.755
		109.197
	140.067	140.067

18″ Rock Elm, 80′ L. of 36 + 50.

+ S.	H. I.	— S.	Elevation.	Objects.	
	109.197	O.K.			
		2.71	106.49	40	
		2.02	107.18	41	
		1.24	107.76	+ 60	
		1.44	107.76	42	
		0.973	108.224	OK BM	Root of

End of these levels.

Continuation of above levels.

+ S.	H. I.	— S.	Elevation.	Objects.	
			108.224	B.M.	above.
0.741	108.965				
		2.58	106.48	40	
		1.76	107.20	41	
		1.16	107.80	42	
		2.80	106.16	+ 35	
		6.68	102.28	+ 55	
		9.82	99.14	+ 80	
		11.424	97.541	43	T. P.
0.749	98.290				
		6.13	92.16	+ 25	
		10.31	87.98	+ 50	
		11.723	86.567	+ 60	T. P.
			O.K.		

Location—Maple Bough Township.

1890-7-14.

F. WALKER, Leveler. S. POKE, Rodman.

Level, Plumbob and Linestaff, 2821.

	Proof.	
	+S	—S
	109.197	0.973
		108.224
W. Pine 38″, 75 ft. R. of 42	109.197	109.197

S. Eye. Level. S. Toe, Rod. Lev.—Blunt, 149

1898-6-1

	Proof.	
	+S	—S
	108.224	
	0.741	
	0.749	11.424
		11.723
Brow of steep descent.		86.567
	109.714	109.714

Juniper Hill and Sunny Grove R. R. Line L.

+ S.	H. I.	— S.	Elevation.	Objects.	
			86.567	+ 60	T. P.
0.521	87.088				
		11.682	75.406	+ 85	T. P.
0.674	76.080				
		8.21	67.87	44	
		11.593	64.487	+ 30	B. M. ×
0.479	64.966				
N. Y. Rod.					
— 6.740		8.21	56.76	+ 35	
+ 6.272		11.462	53.504		T. P.
			1.50		
— 0.468	53.036		52.00	+ 50	
— 6.740		10.335	42.701	+ 70	B. M. ×
+ 0.776					
— 5.964	36.737				
		7.01	29.73	45	
		11.630	25.107		B. M. ×
0.652	25.759				
		2.34	23.42	+ 15	
		5.121	20.638	+ 35	T. P.
0.499	21.137				
		11.311	9.826	+ 50	T. P.
0.501	10.327				
		5.390	4.937	OK BM	
		11.483	—1.156	+ 85	T. P.
0.636	— 0.520				
	O.K.				

Location—Maple Bough Township.

1898-6-1.

S. EYE, Lev. S. TOE, Rod. Lev.—Blunt, 149.

on top of step in face of ledge of syenite on line.

	Proof. + S	− S
Top of stake.	4.937	
	5.390	
	0.636	11.483
	0.520	
Edge of shelf of rock, on line.		
	11.483	11.483

Top of Ledge, 65 ft. L. of Sta. 45. Prominent projection.

Proof. + S	− S
86.567	
0.521	
0.674	11.682
0.479	11.593
	11.462
	0.468
	10.335
	5.964
0.652	11.630
0.499	5.121
0.501	11.311
	5.390
	4.937
89.893	89.893

Root of twisted white pine 100 ft. L. of 45 + 50. No other pine near.

+ S.	H. I.	— S.	Elevation.	Objects.		
	— 0.520					
		10.87	— 11.39	46		
		11.326	— 11.846	+ 10	T. P.	
0.577	— 11.269					
			O.K.			
		3.492	— 14.761	B.M.		×
		11.333	— 22.602	+ 50	T. P.	
0.482	— 22.120					
		11.510	— 33.630	+ 75	T. P.	
0.431	— 33.199					
			O.K.			
		2.277	— 35.476	B.M.		×
		3.33	— 36.53	47		
		4.12	— 37.32	+ 20		
		5.10	— 38.30	+ 80		
Rod.	Target.	3.02	— 36.22	48		
Troy	Upper	2.11	— 35.31	+ 45		
0.615		1.06	— 34.26	49		
0.778		0.782	— 33.981		T. P.	
			1.40			
— 0.163	— 34.144	O.K.	— 35.38	50		
		3.11	— 37.25	+ 60		
		6.26	— 40.40	51		
		9.87	— 44.01	52		
		10.16	— 44.30	53		
		10.04	— 44.18	+ 40		
		7.74	— 41.88	54		

Location—Maple Bough Township.

1898-6-1.

S. EYE, Lev. S. TOE, Rod. Lev.—Blunt, 149.

on step on side of large detached fragment of rock 15 ft. thick, near central part, about 2 ft. above ground, on side next to spring which is 75 ft. L. of 46 + 25.

on step of limestone ledge, in place, about 4 ft. above ground, 75 R. of 46 + 75.

	Proof.	
	+ S	− S
		0.520
	0.577	11.326
Top of stake at Sta. 50.	14.761	3.492
	15.338	15.338

Proof.	
+ S	− S
	0.520
0.577	11.326
0.482	11.333
0.431	11.510
	0.782
	0.163
34.144	
35.634	35.634

Proof.	
+ S	− S
0.782	33.981
35.476	2.277
36.258	36.258

+ S.	H. I.	— S.	Elevation.	Objects.
	— 34.144			
		6.06	— 40.20	+ 65
		5.91	— 40.05	55
New El.		0.222	— 34.366	B.M. ×
	from		O. K.	
Line C.	Line C.			
El.	Datum.	4.37	— 38.51	55 + 37.6 =
712.90	712.70	3.16	— 37.30	269C
712.82	710.64	5.22	— 39.36	270C
709.87	709.75	6.11	— 40.25	271C
707.33	707.17	8.69	— 42.83	272C
708.25	708.09	7.77	— 41.91	273C.

From above B. M. to B. M. on Line C., Root of 40″

0.222	34.366	
3.926	8.471	
4.781	2.216	
1.466	5.311	
	9.987	
750.000		Datum for Line L. above
	700.219	El. B. M. Line C. above noted.
760.395	760.570	
+0.175		Apparent error of these levels.
760.570	760.570	

Location—Maple Bough Township.

1898-6-1.

S. EYE. Lev. S. TOE, Rod. Lev.—Blunt, 149.

on projection of limestone ledge, in place, about 10 ft. above ground, and 150 ft. R. of 55 + 50.

271 +259.3 of Line C.

Proof.

+S	−S
	34.144
34.366	0.222
34.366	34.366

Sycamore 70 ft. L. of 245 + 60 C. on L. bank of Clam Creek.

datum for Line C.

S. Eye, Leveler.

CONTINUOUS LEVEL NOTES.

Juniper Hill and Sunny Grove R. R. **Line L.**

+ S.	H. I.	— S.	Elevation.	Objects.	
			104.632	B.M.	Root of
4.785	109.417				
		5.931	103.486	3 + 45	T. P.
6.421	109.907				
		10.432	99.475	OK BM	Root of
		10.365	99.542	10	T. P.
0.861	100.403				
		11.170	89.233	11 + 50	T. P.
0.417	89.650				
		10.978	78.672		T. P.
0.396	79.068				
		9.294	69.774	11 + 70	T. P.
		8.437	70.631	OK BM	Root of
		10.496	68.572	11 + 95	Edge of

Round soft marsh.

4.633	4.557
3.741	3.817
4.692	4.713
13.066	13.087
79.047	0.021

10.477	68.570	18 + 48	Edge of
0.368	78.679	B. M.	× on a
	O.K.		

RODMAN'S BOOK.

Location—Maple Bough Township.

1890-7-14.

F. WALKER, Leveler. S. POKE, Rodman.

Level, Plumbob and Linestaff, 2821.

a 16″ Beech, 65 ft. Left of 748 × 75, Line B.

a 14″ Hard Maple, 80 ft. R. of 7 + 70 where line leaves woods.

Proof.	104.632		**Proof.**	99.475	
	4.786			10.432	
	6.421	5.931		0.861	10.365
		10.432		0.417	11.170
		99.475		0.396	10.978
	———	———			8.437
	115.838	115.838			70.631
27″ Elm 75′ L. of 11 + 75.				———	———
water in Crystal Lake. Water level.				11.581	111.581

		Proof.
	+ S	— S
	79.068	
	4.633	4.557
	3.741	3.817
	4.692	4.713
		0.368
		78.679
	———	———
	92.134	92.134
water in Crystal Lake. Water level.		

6′ boulder, 2′ out of ground, 80′ R. of Sta. 19.

+ S.	H. I.	— S.	Elevation.		Objects.
			78.679		
9.433	88.112				
		6.657	81.455	OK BM	Root of
		0.896	87.216	20	T. P.
8.472	95.688				
		0.000	95.688	24 + 45	T. P. Peg
10.661	106.349				
		3.627	102.722	OK BM	Top of W.
		0.318	106.031	25 + 35	T. P.
11.014	117.045				
		0.000	117.045	Peg	T. P.
11.026	128.071				
		0.000	128.071	Peg	T. P.
11.262	139.333				
		1.978	137.355	OK BM	Top of
		10.307	129.026	35	T. P.
—0.527	128.499				
		9.281	119.218	OK BM	Root of
0.371	119.589				
		10.755	108.834	OK BM	Root of
0.363	109.197				

Location—Maple Bough Township.

1890-7-14.

F. WALKER, Leveler. S. POKE, Rodman.

Level, Plumbob and Linestaff, 2821.

	Proofs.	
	+ S	— S
10″ Red Oak 75′ L. 19 + 50	78.679	
	9.433	6.657
		81.455
driven to level of cross wire.	———	———
	88.112	88.112
T. stone fdn. S. E. cor. brick dwelling, 200′ R. Sta. 25	81.455	
	6.657	
	8.472	0.896
driven to level of cross wire.	10.661	0.000
		3.627
		102.722
	———	———
	107.245	107.245
cap stone (S. W. cor.) bridge seat, E. end of highway bridge, over Roaring Creek, about 800′ W. of J. Smith's house.	102.722	
	3.627	
	11.014	0.318
Top of stake.	11.026	0.000
	11.262	0.000
18″ Rock Elm, 80′ L. 36 + 50		1.978
		137.355
	———	———
	139.651	139.651

Juniper Hill and Sunny Grove R. R. Line L.

+ S.	H. I.	— S.	Elevation.	Objects.
	109.197			
		0.973	108.224	37 + 70 T. P.

End of these levels.

This line was continued by S. Eye and S. Toe

Location—Maple Bough Township.

1890-7-14.

F. WALKER, Leveler. S. POKE, Rodman.

Level, Plumbob and Linestaff, 2821.

38″ White Pine, 75′ R. Sta. 42

Proof.

+S	−S
119.218	
0.371	
0.363	10.755
	0.973
	108.224
119.952	119.952

Proof

+S	−S
137.355	
1.978	
	10.307
	0.527
	9.281
	119.218
139.333	139.333

in 1898. See next page.

Juniper Hill and Sunny Grove R. R. Line L.

+ S.	H. I.	— S.	Elevation.	Objects.
			108.224	B.M. p. 42.
0.741	108.965			
		11.424	97.541	43 T. P.
0.749	98.290			
		11.723	86.567	43 + 60 T. P.
0.521	87.088			
		11.682	75.406	43 + 85 T. P.
0.674	76.080			
		11.593	64.487	44 + 30 B. M. ×
0.479	64.966		O.K.	
— 6.740		11.462	53.504	T. P.
+ 6.272				
— 0.468	53.036			
— 6.740		10.335	42.701	44 + 70 B. M. ×
+ 0.776			O.K.	
— 5.964	36.737			
		11.630	25.107	B. M. × top of
0.652	25.759		O.K.	
		5.121	20.638	45 + 35 T. P.
0.499	21.137			
		11.311	9.826	45 + 50 T. P.
0.501	10.327			
		5.390	4.937	OK BM Root of
		11.483	—1.156	45 + 85 T. P.
			O.K.	

Location—Maple Bough Township.

1898-6-1.

S. EYE, Lev. S. TOE, Rod. Lev.—Blunt, 149.

Proof.

+S	−S
25.107	5.121
0.652	11.311
0.499	5.390
0.501	4.937
26.759	26.759

	Proofs. +S	−S
Brow of steep descent.	108.224	
	0.741	
	0.749	11.424
	0.521	11.723
	0.674	11.682
		11.593
		64.487
top of step in syenite ledge on line.	———	———
	110.909	110.909
Top of stake.		
	64.487	
	0.479	
edge of shelf of rock, on line		11.462
		0.468
		10.335
ledge 65′ L. of Sta. 45. Prominent projection.		42.701
	———	———
	64.966	64.966
	42.701	5.964
		11.630
		25.107
	———	———
twisted white pine 100′ L. 45 + 50. No other pine near.	42.701	42.701

+S.	**Proof.** −S
4.937	11.483
5.390	− 1.156
———	———
10.327	10.327

Juniper Hill and Sunny Grove R. R. **Line L.**

+ S.	H. I.	— S.	Elevation.	Objects.
			— 1.156	T. P. last
c.636	— 0.520			
		11.326	— 11.846	46 + 10 T. P.
0.577	— 11.269			
		3.492	— 14.761	OK BM × on
		11.333	— 22.602	46 + 50 T. P.
0.482	— 22.120			
		11.510	— 33.630	46 + 75 T. P.
0.431	— 33.199			
		2.277	— 35.476	OK BM × or.
0.615				
0.778		0.782	— 33.981	T. P.
— 0.163	— 34.144			
		0.222	— 34.366	OK BM × on

For connection with B. M. Line C. see next page.

Location—Maple Bough Township.

1898-6-1.

S. EYE, Lev. S. TOE, Rod. Lev.—Blunt, 149

page.	**Proofs.**	
	+ S	— S
	0.636	1.156
	0.577	11.326
step or side of large detached fragment of rock 15′ thick, near middle, about 2′ above ground on side next to spring, which is 75′ L. 46 + 25.	14.761	3.492
	15.974	15.974
	3.492	14.761
step of limestone ledge, about 4′ above ground, 75′ R. of 46 + 75.	0.482	11.333
	0.431	11.510
	35.476	2.277
Top of stake, Sta. 50		
	39.881	39.881
projection of limestone ledge, about 10′ above ground, 150′ R. of 55 + 50	2.277	35.476
		0.782
		0.163
	34.366	0.222
	36.643	36.643

Connection of B.M. 150′ R. Sta.
B.M. 70′ L. Sta.

+S	−S	
0.222	34.366	B. M. Line L. p. 44.
3.926	8.471	
4.781	2.216	
1.466	5.311	
	9.987	
	700.219	El. B. M. Line C. Root of 40″
750.000		Datum for Line L. Above datum
760.395	760.570	
+0.175		Apparent error.
760.570	760.570	

Location—Maple Bough Township.

1898-6-1.

S. EYE, Lev. S. TOE, Rod. Lev.—Blunt, 149.

55 + 50 Line L, with
245 + 60 Line C.

Sycamore 70′ L. 245 + 60 C. on L. bank of Clam Creek.
for Line C.

S. Toe, Rodman.

7. The leveler and rodman should not pass each other, in working, without agreeing upon the elevation of the last turning point, or height of instrument, as the case may be. Whenever a B. M. is established they should also agree as to its elevation. When using a target rod, the rodman's record of readings on turning points and benches, will be found of much service in windy weather. When using a speaking rod the leveler gives the rodman all readings on turning points and benches.

8. Field Work of Running Levels. Set up the level in the same general direction, if possible, from the starting bench that the survey is to proceed in; where the rod held on this bench can be read; but not over 350 feet away from the bench, for the best instruments and work. Hold the rod on the bench and set the target. Clamp the target. If the rod reading is over five feet, wave the rod to and from the level, past a vertical, both ways. Finally clamp the target so its sight line will just coincide with the elevation of the horizontal wire in the level when at its highest point if waved as above directed. Take frequent notice that the level bubble is in the right place. After one final look at the bubble signal the rodman to read the rod. Set this reading down in the first or left hand column of the left hand page in each note book, on the line next below that one, in the fourth column, on which stands the elevation of the bench. Call the reading back to the rodman as written down and see that he gives a return signal that it is called correctly according to his record. (See specimen notes above for examples showing the records.) Add this rod reading to the elevation of the bench for the height of instrument above datum, which enter in the second column on the same line with the rod reading (or plus sight) that gave it. Take the rod to the first stake. Look at its number. Call the number to the leveler. Hold up the rod on the ground beside the stake, and take a reading there, simply waiting to set the target without clamping it and waving the rod. As the rodman starts for the next stake he should call this reading to the leveler who should call it back and receive the rodman's signal that he has it right. Set this reading down in the level book in the third column on the next line below that on which

stands the height of instrument in the second, with the number of the stake opposite, in the fifth column. While the rodman is going to the next stake subtract this rod reading (or minus sight) from the height of instrument and enter the corresponding elevation, in the fourth column, on the same line with the rod reading and stake number. See that the level is ready for the next sight. Repeat these operations till the rodman comes to, or opposite the level. Compare records as to the height of instrument, and there agree upon it. Go on as before till the rodman is far enough away, or on such ground, that it is necessary to move the level forward. Drive a peg in the ground, on the line of the stakes, till its top is even with the surface. Place this peg where an elevation is needed, or will be useful, if possible. Use a peg long and large enough to form a firm seat for the rod. Take an accurate rod reading on top of this peg with the same care as if it was a B. M. Call this reading and record it in the third column, before taking up the level. Subtract it from the height of instrument above in full and enter the elevation of the peg in the fourth column on the line with this minus sight. Enter in the fifth column the letters T. P. (for turning point) and the number of the stake or plus on the line. Take up the level. Go with it to the rodman and agree with him as to the elevation of the peg. Go on and set up the level beyond the peg at a suitable place not over 350 feet away from it for the conditions first stated. Take a rod reading on the peg as if it was a bench (which it temporarily is). Set this in the plus sight column on the line below the elevation of the peg. Add this plus sight to the elevation of the peg for the new height of instrument, which enter in the second column on the line with its plus sight. Go on with the levels as during the first setting of the level. Repeat these operations, here set forth, at will.

9. **Preliminary Levels.** In running preliminary or exploration lines the stakes may be set at any convenient interval, from 100 to 500 feet being usual, depending upon the character of the ground over which the line runs. Where the stakes are 500 feet apart, and the country open, the leveler and rodman should both be mounted. They should take from 1,500 to

3,000 feet of line at each setting and the rodman must keep all the level notes. The pegs for turning points should be set equally distant each way from the level by setting the level each time about half way between two stakes and driving the pegs at stakes. On such work the rod is set on the ground beside the stakes and read to the nearest tenth of a foot, except on turning points and benches, where it should be read to hundredths, and with care. The factor of precision to be expected on such work must correspond to the character of it. In ordinary work where the stakes are spaced at any distance up to 200 feet a B. M. should be put in about every 2,000 feet, and on both sides of any obstruction to leveling, or to travel, as a body of water, a morass, or a cliff. On steep slopes bench marks should be put in so often that there should not be over forty feet difference in elevation between any two successive ones, however near they may be to each other. There should be one near the top and another near the bottom of the slope. The amount of other notes besides the actual levels that are taken must depend on the special requirements that are made upon the leveler by his employer in any given case, and upon the time he has to make them in. It is as well here, as elsewhere, to record every important fact which it is reasonably possible to get. There is a great difference in the capacities of men in this respect.

10. Location Levels. In this work the stakes should be set not over 100 feet apart and each one centered after driving and the distance to it tested. The rod should be set on a stout peg driven with its top even with the surface of the ground at each stake and read to hundredths. These level pegs should customarily be set in the same relative position with reference to the stakes so as to be readily found afterwards. They are sometimes set about two-tenths of a foot from the numbered side of a stake and on the line. They should be driven so the elevation of their tops will farily represent the elevation of the surface of the ground regardless of its minor inequalities. Benches should be placed not over 1,000 feet apart; on both sides of obstructions; at the tops and bottoms of all considerable banks or steep slopes; and for each 20 feet difference in elevation on steep slopes. They should be placed so levels

can be "given" anywhere along the line at the first "set up." There is no time when a B. M. can as cheaply be put in where it is wanted as when the levels are first run. All the useful information, practicable, should be recorded.

11. **Levels in general.** Levels are run to enable a plat of a section of the surface of the ground, that is a profile, to be made. From thence the levels are made the basis for the cross-sections, the computation of the earthwork, and the pay for it. The more accurately the profile, or the levels from which it is made, record the inequalities of the surface, the greater the precision that can be attained in the subsequent work. In running the levels a rod reading should be taken at every prominent change in the surface of the ground along on the line,—at the breaks of the small ridges, at the bottoms of the corresponding depressions, at the edges of all banks, top and bottom, the edges of both tops and bottoms of channel ways of streams, however small, and all such places. Points thus noted may be between two stakes. The distance, in each case, from the stake bearing the lesser number should be paced or otherwise estimated by the rodman and called out to the leveler in the same manner as the number of a regular stake should be at the time he is setting up his rod for a reading. Such distances are called plusses and are so entered in the level book in feet. (See specimen notes above). On preliminary work read the rod to tenths on the ground at the plusses. On location drive a peg at every plus, exactly on the line, estimate the plus distance, as above directed, and read the rod to hundredths the same as at a stake. On location readings should be taken so often that the surface of the ground may be regarded as straight between the places where the rod is set to get them. It should always be the aim of the rodman to locate his turning points so they will work in usefully in making up the profile, that is they should be at some stake or plus where an elevation is needed. Whenever any water is crossed get the elevation of it. If the body of water is large get the elevation at both edges of it. Do not in any case use the water as a turning point for the purpose of transferring the levels from one side of it to the other as across a mill pond. Water is not level as often as it is assumed to be.

Especially do nothing of this kind in a swamp or marsh as it will be an accident if the water is precisely the same level on both sides of it. To take a water elevation drive down a peg in the bottom till its top is even with the top of the water and set the rod on that. Notes should be made in passing of nearby objects that would affect the changing of the line either favorably or unfavorably. Such objects are the bends of rivers, ponds, lakes, swamps, high banks, ridges, steep slopes, ledges, or smooth flat bottoms, high plateau land, a beach, shallow water, firm ground. In making notes of such objects be definite in the record. Give the distance from some place on the present line to the object and the difference in elevation as nearly as it can be ascertained. Sketch any possible outlines but record the heights and distances in writing or by means of the scale adopted on the sketch, or right hand page, or by contours, giving a note of this scale and the difference in elevation between the successive contours. Contours seem to furnish the most desirable method of sketching ridges, bluffs, and ravines. In noting an elevation beside the line state the rise or fall for each given distance from the line. In making notes of this kind the facts recorded must be ascertained by the best means at hand. If there is nothing better the whole must be estimated. The judgment, by experience, will become very trustworthy, in some persons, in such matters. These things properly belong to the topographer on a survey, but frequently no such person is necessary and the leveler may have much of it to do.

Beginners in leveling mix their notes. Keep the notes in prescribed form,—whatever it may be,—in every particular, omitting nothing, adding nothing, changing nothing. A novice is not the one to prepare the forms for use on any work.

Make the numbers continuous, as they are on the ground, —no skips. The rodman should be on the lookout all the time for stakes that are numbered wrong. The leveler may assist materially by watching the numbers running in order on the page of his note book.

The rod is read wrong. The leveler can prevent errors of even feet in readings by observing the positions of the even feet marks on the rod with respect to the body of the

rodman. He should also know the distance from the bottom of the up sliding part of the rod to the target when using "long rod." By noticing whereabouts the bottom of this part of the rod comes he can tell the "long rod" reading to the nearest foot and so prevent errors in the feet of "long rod" readings. The lower target on the Troy Rod will answer the same purpose, better.

Additions and subtractions are made wrong. Ten feet, one foot, and one-tenth of a foot errors are common. "Prove" every page of level notes. Both leveler and rodman "prove" every day's work and agree upon the figures, before beginning another day's work. This should be done on the work at quitting time.

Elevations are not figured out. This is simple neglect and on all ordinary work is inexcusable. When the day's work is done and "proved" the leveler's book should be ready to "make out" the profile from.

Titles, and headings, are omitted. Inexcusable!

Dates are forgotten,—not entered. Plain neglect!

Names of party are omitted. Heedlessness!

Stations, and other places where elevations are needed, are skipped. Inattention!

Always get the elevation of the surface of all water crossed by the line, and the depth of the water, where practicable.

Channel banks of streams, and ditches are omitted. Get elevations at top and bottom of all such no matter how close together the places are.

The same places are omitted in crossing highways.

Without such data, no proper drawing of a bank, a ditch, a channel, or a highway crossing, can be made. A profile is a drawing to scale and should be correct.

Take levels on two or more places on any previous work from which the new work is to proceed.

Take levels on two or more places on any existing work with which the survey is to connect.

Do not be afraid of getting too much information, or of carrying the levels too far back, or ahead.

Check on all accessible bench marks, water surfaces, or other objects, that will connect the new survey with previous

work. When doing so give both the old and the new elevations and show their differences.

Do not think of calling yourself a leveler, or rodman, till you habitually do all of these things.

12. It is customary to "**Make out**" a profile of each day's levels as the work proceeds. This is done to keep track of the ground covered, to see if the line is satisfactory, and to suggest needed changes. The final profile, upon which construction is to be based, should not be made out till after check levels have been run to verify the elevations of all bench marks along the line.

13. **Check levels** are peg levels, run from each bench mark to the next showing the difference in elevation between them. These levels should be run with extra care during favorable weather. Be particular to keep the level in good adjustment. Avoid every thing likely to introduce errors into this work. Leveler and rodman both keep notes, and reduce them separately and independently. Useful forms are given below. Start from a bench whose elevation is to be trusted, or correct all elevations to agree with the elevation of the trusted bench after the check levels are finished. The elevations of benches determined by the check levels are to be taken as the final elevations thereof. All levels along the line are to be corrected to agree with the final elevations of the benches between which they were taken. This may involve rerunning the levels along the line between two benches where the error found indicates a blunder such as is shown between the first two benches in the notes below. This is necessary in order to find out which of the elevations along the line are affected by the blunder, and which are not.

In case the difference in elevation between any two benches found by the check levels differs from the original difference in elevation between them so much as to show an avoidable, or inadmissible, error in the work, repeat the check levels between those benches until the differences in elevation between them, obtained by the check levels, agree sufficiently well. Take the half sum of these differences as the correct difference in elevation between those benches. Run back from

the forward bench on the same pegs used in running forward. By so doing an additional verification of the work can be found by computing the successive differences in elevation between pegs and comparing them on the forward and backward run. This is shown in the case of the first two benches in the notes. In the case of the second two benches the difference in elevation by the check levels agrees sufficiently well with the original difference in elevation, and no backward run was made. The levels along the line between the first two benches were rerun to locate the blunder. The levels along the line between the second two benches were corrected by adding —0.440 to all the elevations. These are samples of but two of many conditions that may arise. Sometimes considerable work is required to locate, and correct, errors, and blunders in leveling. However, this must be done as the correctness of the work, and the pay for it, depends upon the levels. This is some explanation of the necessity for painstaking, and trustworthy, work in running the levels at first.

PROFILE.

14. **A profile** is a plat of a line of continuous levels. To make a profile, plat the elevations of the stations and plusses given in the level notes, to a scale, at their proper distances apart and join the points thus obtained on the paper by a fine line drawn free hand. This line is called the Surface Line. The scales are usually different for the vertical and horizontal measurements. The vertical is made the larger in order to exhibit the differences in elevation with greater precision, as these are the more important. Such a plat may be made on any kind of paper but it is customary to use some of the specially ruled papers prepared for this purpose. There are many varieties of this now furnished by dealers, printed in red, green, or blue. It is furnished in sheets or continuous, with cloth back, or without, ruled by inches or by meters. There are three plates in common use, called Plate A, Plate B and Plate C. Plate A is ruled four spaces to the inch horizontally and twenty vertically, every fifth and fiftieth horizontal line prominent. Plate B is four horizontal spaces to the inch, thirty vertical, and has every fifth and twenty-fifth horizontal line prominent. Plate C is five horizontal spaces to the inch, twenty-five vertical, and has every fifth vertical line and every fifth and twenty-fifth horizontal line prominent. As an illustration of the use of Plate A, it is customary to call each horizontal division one hundred feet and each vertical division one foot. The horizontal scale is then 400 feet to the inch and the vertical scale 20 feet to the inch. The exaggeration is twenty, that is, the differences of level show twenty times their comparative value. In platting the elevations, or "Making out" the profile, as it is called, it will be evident that a strip of paper is needed no wider than is sufficient to receive the actual total difference in elevation between the lowest and highest points to be platted. For this reason it is customary to assign an elevation to the lowest heavy horizontal line on the strip to be used of such a multiple of five feet as will bring it below

the lowest point to be platted and make the count upwards to the still heavier lines such as to bring them at some multiple of twenty-five feet, as 50′, 100′, 175′, 625′, etc. In cases where the actual difference of elevation would require a paper so wide as to be inconvenient it is usual to plat the points on a strip of suitable width until they would run off and then assume new available elevations for the counting lines and begin again. This change may be repeated as often as necessary. Such profiles are not always desirable for office use. When this is the case strips may be made up of any desired width by joining one to another in the manner described below. Having settled upon the elevations of the counting, or heavy, lines, these elevations should be written in penciled figures at both ends of every separate piece of profile, though, on the same strip, and besides at every hundred station vertical line. In case a piece of paper is not long or wide enough another may be joined to it as follows:

15. **Splicing Profile Paper.** Choose the line on each piece that is to form the junction of the pieces so that the count of lines will be continuous when the pieces are joined with at least a quarter of an inch of lap to each piece. With a sharp knife cut very slightly into the paper from the ruled side on each piece, along the junction line on each. Prick a hole through each piece on each end of a line one-quarter of an inch back as the count is to run on the joined paper. Turn both pieces over and with a straight edge cut a line very slightly into the back of the paper, the same as was cut on the ruled side, joining the prick marks on each piece. With the sharp knife peel up the paper at the end of the cut on the ruled side and split the paper back to the cut on the back side. With the hands the split may be extended across the paper, splitting it evenly from one cut to the other, by a little care in drawing the pieces asunder. A few trials will enable almost anyone to succeed well. When both pieces have been thus split, and scraped a little in places it may be, each will have a thinned beveled edge of the proper form to join with the other neatly. Next fasten the pieces to a board so the junction lines will coincide nicely and the rulings range well, leaving an inch or two of the upper lapping piece free to be

raised. Raise the top lap and apply a little mucilage, glue, or paste, neatly and evenly to the splice laps. Join them carefully. See that the line of union is perfect and let them dry till ready for use. In fastening the pieces to a board get the lines at right angles to the union line exactly matched and set them truly in range by a long straight edge, or thread. This must be done if the united pieces roll smoothly.

16. **"Making out" a Profile.** Number from left to right, the station lines as needed, and write the proper elevations on the horizontal lines. As the number of a station and its elevation are read from the level notes find the station and repeat it back to the reader to make sure it is right; count up the elevation (estimating the tenths of a foot), make a pencil dot at the place on the station line and call the elevation as noted back to the reader. When the point for the next station is fixed in the same manner just draw the pencil back to the last point. Thus proceed at will. Estimate the horizontal distance on the paper for the plusses between the regular station lines.

A little experience will enable a qualified person to plat regular ground as fast as the elevations are slowly read off. It is customary to omit the reading and calling of the regular stations except at the even tens. Very few errors are made by experienced men in thus making profile. The reader needs to look out for the benches, and turning points that are not at stations, in this practice, and not read then as part of the profile. Some levelers, for this cause, keep the elevations of such in a separate column. This seems hardly necessary in the form of notes above given. After the profile is in pencil it may be inked and the numbers on it written in ink. Any important information may be added in the open spaces on the paper referring each item to its proper station by number. In this way a profile should show the names of all objects of interest, such as towns, streams, roads, and so forth. It may also show where clearing begins and ends, the location of objects near the line, and in short any matter that may be made available either in construction or for purposes of reference. As profiles for field use are very commonly made on a narrow strip of paper, this will be found a very

convenient form to carry a record of the information obtained on the survey to which the profile belongs. Profiles made out in full, as here noted, are usually not needed except for the located lines. As work is very generally considered in sections of about a mile each it is best to make these sections 5,000 feet long and note the division points on the profile by some plain device. Many use 5,300 feet for a section.

17. **Grade Line.** While the profile is yet in pencil or has but its surface line inked, the grade line may be drawn on it in pencil. This is the grand object for which the profile was obtained,—the establishing of the necessary grade. The grade line represents the surface of the finished work as the surface line represents the surface of the ground. On railroad work there are three noticable grades. One is the surface of the top of the rails at the center of the track. Another is the surface of the top of the ties at the center of the track, which is commonly the surface of the ballast into which the ties are bedded. The third is the surface of the line at the center of the track before the ballast is added, and is the surface left by the graders. It is this last and lowest grade line that is to be considered here.

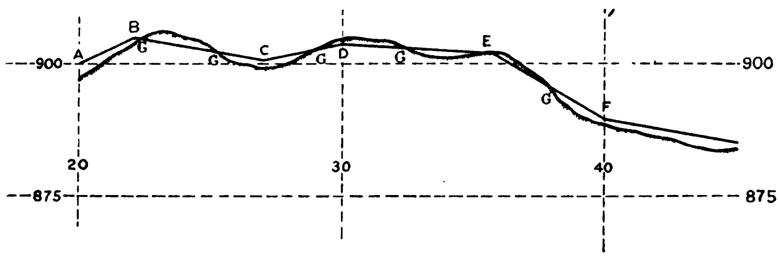


Fig.1.

18. **Drawing a Grade Line on a Profile.** Provide a fine black thread and some needles. Spread out and fasten the profile upon a board. Fasten one end of the thread near one end of the profile. Stretch the thread over the profile with one hand and by sticking the needles through the paper into the board with the other, fit the black thread so as to represent the required grade line. By keeping a little tension on

the thread with the hand holding the free end, it may be made to show a good line on the paper. Shift the needles as study and experience direct till the thread covers the desired line. In setting the needles it is usual to fix them at easily designated points, both horizontally and vertically. Having placed the needles where it is desirable to leave them, remove the thread and connect the points where the needles stand by straight lines. Note these points both as to horizontal distance and elevation.

The points where the needles stood are where the inclination of the grade changes. They are grade vertices. It is from their elevations and horizontal distances apart that the rates of grade and thence the elevations of grade at all places on the line are obtained.

The grade line should be inked. The station numbers and elevations of all the grade vertices should be inked. The rates of the inclinations of the straight pieces of the grade line and the change in elevation per station distance should be inked on each piece, when found as directed below.

19. **For getting elevations of grade,** subtract the station number of the grade vertex at the left hand end of a straight piece of the grade line, from the station number of the grade vertex at the right hand end of it. Should either one, or both, of these grade vertices be at a plus station, reckon the plus in decimals of a station distance. Subtract the elevation of the left hand end from the elevation of the right hand end, noting the sign, plus for an ascent, minus for a descent. Preserve and use this sign. Divide this difference in elevation by this number of station distances to get the change in elevation per station distance, noting the sign. Divide this change in elevation per station distance by the length of the station distance to get the rate of grade, noting the sign, which is also the tangent of the angle which the grade line makes with a level line.

If the grade vertices are at stations, add, by its sign, the station change to the elevation of the left hand grade vertex, as taken from the profile, to get the elevation of grade for the next regular station. Continue to add, station by station,

numbering the results in order, till the elevation for the right hand grade vertex is reached. See if this is the same as that given by the profile. If not, review the work and correct the errors.

If the left hand vertex is at a plus, compute the change in elevation for the rest of the distance forward to the next regular station by multiplying this distance by the rate of grade, as found above, noting the sign. Add, by its sign, this computed change in elevation to the elevation of the left hand grade vertex to get the elevation of grade for the succeeding regular station. Then add, by its sign, the station change in succession for each regular station, numbering the results in order, to the station at the right hand grade vertex, if this is at a station, if not, stop at the regular station next preceeding. Compute the change for the plus to the right hand grade vertex, using the rate and distance, as above. Add this change to the elevation of grade for the last station to get the elevation of grade for the right hand grade vertex. See if the elevation of grade for the right hand grade vertex as computed is the same as that given by the profile. If not, review the work and correct the errors.

Ex. Stations 100 feet apart.

148 + 75 Number of left hand grade vertex.
165 + 50 Number of right hand grade vertex.

In decimals of a station distance, these would be 148.75 and 165.50. 165.50 — 148.75 = 16.75, station distances on this piece of grade line.

432.50 Elevation of left hand grade vertex.
417.00 Elevation of right hand grade vertex.
16.75) — 15.50 Difference in elevation.
100) — 0.9254 Change in elevation per station distance.
— 0.009254 Rate of grade.

25 × — 0.009254 = — 0.23135. Change for 25 feet.
50 × — 0.009254 = — 0.46270. Change for 50 feet.

STA.	EL. GD'E	STA.	EL. G'DE	STA.	EL. GD'E
148 + 75	432.50	155	426.72	165	417.47
	— 0.23	6	425.79		— 0.46
149	432.27	7	424.87	165 + 50	417.01
	— 0.93	8	423.94		
150	431.34	9	423.02		
	— 0.92	160	422.09		
1	430.42	1	421.17		
2	429.49	2	420.24		
3	428.57	3	419.32		
4	427.64	4	418.39		

The elevations of grade, as above calculated, are to be found for each straight piece of grade line throughout the work.

20. At the grade vertices the change in rate from one stretch of grade to the next may be so great as to require a vertical curve to connect the two straight lines. The parabola is used for this purpose. It is assumed to be tangent to the straight grade lines three or four stations each way from the grade vertex. Elevations are computed for the points where vertical lines at the stations intersect this curve. The grade line between successive stations on the curve, is usually taken to be straight, that is a chord of the parabola.

Any equal spacing may be used in place of the station spacing, so that the tangent points are taken, each at the same number of these spaces from the grade vertex.

In very smooth work, like stone walks, round off all changes of grade, or they will show in the finished work. This is especially true where the grade vertex is convex downwards.

21. In Rounding Off Grades, let,

$n =$ the number of station distances (or other equal divisions of the grade line) each way from the grade vertex, V, to the tangent points, T, of the required parabolic curve; the values of g and g', below, being computed with respect to the horizontal spacing of these equal divisions of the grade line.

$\pm g =$ the rise, or fall, (+ rise, — fall) of the grade line for each station distance, or other equal horizontal division, between T and V, for the line approaching the grade vertex, V, from the left.

$\pm g' =$ the rise, or fall, (+ rise, — fall) of the grade line for each station distance, or other equal horizontal division, between V and T, for the line leaving the grade vertex, V, to the right.

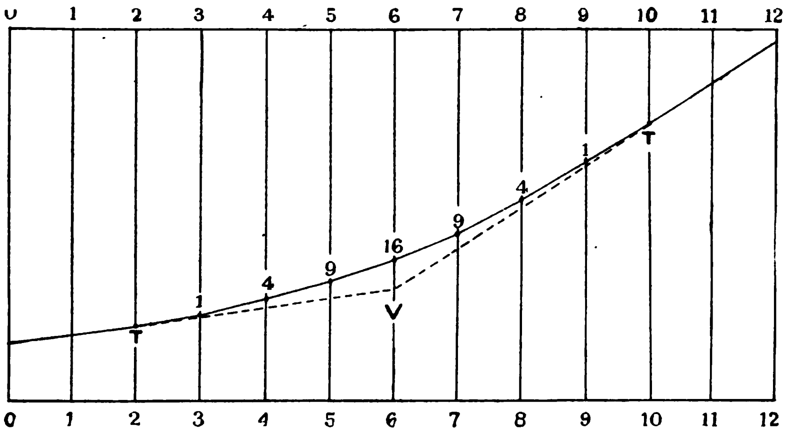


Fig 2.

$\pm t =$ the vertical distance from the straight grade lines (+ up, — down) to the first required points on the parabola from its tangent points, T.

Assume n. Find $\pm t$ from the equation below. Observe all signs.

$$\pm t = \frac{g' - g}{4n}$$

Add, by its sign, t to the straight line elevations of the first stations from the tangent points, T, $4t$ to the elevations of the second stations from the tangent points, $9t$ to those for the third stations, $16t$ for the fourth, and so on, following the squares of the natural numbers.

There are three instances in which the curve lies above the straight grade lines, as in the figure, and three others in which the curve lies below them. Careful attention to the signs, as above directed, will prevent mistakes.

22. Ex. The grade in the last example which has a change of $-0'.9254$ per station distance of 100 feet is followed by a grade with a change of $+2'.25$ per 100 feet. As the grade vertex is at Sta. 165 + 50 it will be well to use half station distances for the equal horizontal spacing of the divisions of the grade line. The changes for half a station distance will be $-0.'4627$ and $+1'.125$. Therefore $g = -0'.4627$ and $g' = + 1'.125$. Assume the tangent points for the parabola to be 350 feet each way from the grade vertex. There will be seven divisions on each tangent, or $n = 7$.

$$t = \frac{+1.125 - (-0.4627)}{4 \times 7} = +0.056704.$$

Desiring the elevations of grade only at the regular stations, or at the even numbered divisions of the grade line, only the even numbers from 0 to 7 are used in the computations.

Sta.	Nos.	Ter's.	To be added.	Elevations.	Cur. El.	Proof. 1st diff.	Proof. 2d diff.
162.	0	0	0.00	420.24	420.24		
						−0.69	
3.	2	$4t$	+0.226816	419.32	419.55		+0.44
						−0.25	
4.	4	$16t$	+0.907264	418.39	419.30		+0.46
						+0.21	
5.	6	$36t$	+2.041344	417.47	419.51		+0.45
						+0.66	
6.	6	$36t$	+2.041344	418.13	420.17		+0.46
						+1.12	
7.	4	$16t$	+0.907264	420.38	421.29		+0.45
						+1.57	
8.	2	$4t$	+0.226816	422.63	422.86		+0.45
						+2.02	
169.	0	0	0.00	424.88	422.88		

Verify the final elevations by finding their second difference. This should be $8t = +0.4536$, nearly. If the elevations had been worked out for the whole number of points the second difference would be $2t$ instead of $8t$. In general, if m points are skipped in spacing the positions for the elevations to be figured, the second difference of the computed elevations will be $2(m+1)^2 t$. In the above case one point was skipped as the spacing proceeded; $m = 1$, and $2(m+1)^2 t = 8t$.

FIELD DATA.

The final elevations of grade, obtained and checked by the processes above set forth, are for use in the field. Usually it is sufficient to take the grade as straight between stations, as was mentioned above, and figure any elevations required for plusses on this basis. In case it ever is necessary to obtain the elevations of points on the curve between stations, it may be done for equal parts of the station distance by assuming n at the total number of all of these equal divisions between the grade vertex, V, and the tangent points, T. Then proceed with the calculations in the regular way, as though these equal divisions were station distances.

The change in elevation of grade between two stations, or other points, on the curve, should be figured by subtracting the elevation at the left hand one from that at the right hand one. Preserve the sign of the result. This is to be preferred to any system of calculation, as the elevations have all been checked.

23. **Field Data.** When the field work of surveying the cross sections of the work is going on, the elevations of the ground along the center line should be verified by rerunning the levels. The elevations of the bench marks were verified by the check levels. The elevations of grade were verified by the profile, and when computing them. The elevations of the level pegs along the line remain to be tested when cross sectioning. This is provided for in preparing the data for field use. A common $4\frac{1}{2}'' \times 7\frac{1}{2}''$ field note book can be used. The following form of record can be used, embracing 7 columns, the seventh being on the right hand page of the book. The first five columns are those of a level book. The sixth col-

1	2	3	4	5	6	7
				Objects		
+ S.	H. I.	— S.	El.	Sta.	Old El.	Grade

umn carries the elevations given by the location level notes. The seventh column carries the elevations of grade as they finally appear after the vertical curves are added for rounding off the vertices. Certain precautions should be observed in preparing this book for field use.

Copy into column five, from the location level notes all the station numbers, and plusses, increasing from the bottom of the page upward, noting all bench marks at their proper places and carrying out the descriptions of them on the right hand page. Copy into the sixth column the elevations given in the location level notes for all the places noted in the fifth column. Verify the copy by comparing it with the original. Copy into the seventh column the final elevations of grade as computed and checked, writing them against the proper station number entered in the fifth column. Verify the copy by comparison with the original. Note on the vertical line at the side of the seventh column; the rate of grade; its change in elevation per station distance; the points between which these quantities apply; the point, T, where the rounding off curve begins; the change of grade per station on the curve; the point, V, where the grade vertex, V, comes; the point, T, where the rounding off curve ends; and the rate on the following straight grade. Continue the record repeating this data along the whole line.

The leveling for checking the elevations given in the sixth column cannot proceed in just the same form as if these levels were being run independently, the first time. The notes for working out the heights of instrument, and the elevations of turning points, which do not come at places noted in the fifth column, must be distributed in the first two columns so as to have any given height of instrument precede the rod readings in the third column which are to be subtracted from it. If the first two columns do not furnish enough room for these notes they can be spread on the right hand page. As the new elevations are worked out they should be entered in the fourth column and compared with those in the sixth column. If they do not agree as well as they should, verify the work, and finally determine the correct elevation to be used thereafter. In this manner fix upon the correct elevation to

be used at any place on the center line before beginning to survey the cross section there.

In this manner, errors in the elevations of the center level pegs can be found and corrected. Otherwise there is slight chance of finding them until the work of grading is done. As the pay for the work is based upon the cross sections, an error only discovered after the grading is done should cause much annoyance, and may make trouble.

STAKING OUT.

24. **Staking Out** is the work of setting and marking the stakes for the guidance of the workmen; surveying the cross sections of the formation; and making such other measurements, observations, and records, as may be required for a trustworthy calculation of the volume of materials in the formation. The stakes needed by the workmen are mainly the center stakes, C, the sides stakes, P, and the grade stakes, G,—see diagrams in advance. C and P at regular stations are usually enough, cross sections at plusses not being marked by stakes, ordinarily. Places between C and P are often needed in computing, but need not be marked for the workmen. Grade stakes, G, come at the places where the road bed (so called) intersects the surface of the ground,—that is where a cut joins a fill,—either along the center line of the formation, or on a cross section that is part cut and part fill, as on a hill side. The surveys are for the purpose of determining all these various points both as to horizontal position and elevation, and for verifying the elevations of the center level pegs, as before indicated. In leveling on the center line elevations were required often enough so that straight lines joining the places where the rod was set on the ground to get them, would fit its surface sufficiently well for the purposes of the work. The same idea rules the work of staking out. If a straight line from the top of the level peg at C to the ground at P will not fit the surface of the ground well, places between C and P must be determined so that a broken line from C to the first place, thence to the next, and so on out to P will fit. If it is found that levels enough were not taken on the center line, this defect must now be made good and all the plusses, new and old, measured from the stake of the smaller number to the plus peg. If straight lines from the ground at P to the same side stake, P, on the next cross section, either way, do not fit the surface of the ground well enough, take one, or more, new cross sections, so placed that the "side lines" of the prismoids between the successive cross sections will fit

the ground well. The prismoid of the formation, that is the volume composed of the materials between two successive cross sections, must be bounded by straight lines only, for its content to be calculated. These lines need not fit the surface of the ground with great nicety, but they must bound, and define, a mass that fairly represents the volume of the prismoid named above. The cross sections may be separated somewhat by taking advantage of the inequalities of the surface of the ground which run about in the direction of the formation. This is done by placing the cross sections so that surface lines may connect the marks on one cross section with those on the next and divide the ground surface of the prismoid between them into a succession of plane triangles, extending from one side line of the prismoid to the other, and having their bases on either cross section while their vertices are on the other. These surface lines must be recorded when the cross sections are surveyed.

Cross Sections, with points on each, should be taken so often that straight lines joining the places where the rod is set in taking them, on the same cross section, and from each one to the next, will fit the surface of the ground well enough for the purposes of the work.

Where a Correction on Curves is needed in figuring the volumes of the formation, take a cross section at the P. C. and P. T. and at the P. C. C. on a compound curve, or the P. R. C. on a reversed curve.

The Tools, and Instruments, used in taking cross sections are Levels, Rods, Tapes, Axe, and Wooden Pins, together with Note Books, Pencils, Erasers, and Stake Marker.

A light Wye Level, or a light Transit with Telescope Level, and a Hand Level, are used. The Wye Level should Transit with Telescope Level is sufficient for taking cross sections. The Hand Level is useful around grade points, and on tions. The Hand Level, is useful round grade points, and on some cross section work, when properly supported. A water proof-cover bag will prevent the transit, or level, from getting wet.

The Leveling Rod. One of the usual patterns, should be used in verifying the elevations of the center level pegs. For

taking cross sections a piece of metallic tape twelve feet long, —or longer, on rough ground,—tacked on a light strip of wood, is sufficient on a great deal of work. Mark the even feet on the strip of wood from a steel tape, and tack the metallic tape to fit these marks. Such a rod can be used at times to measure horizontally with convenience. The Bean Leveling Rod (the invention, many years ago, of H. F. Bean of Jackson, Mich.) consists of a tape with the ends fastened together running like a belt on spools, or rollers, near each end of a wooden rod. The tape may be 20 feet long giving a rod about 10 feet long, or the tape and rod may be of any length desired. This is the most useful rod for taking cross sections. The same principle when embodied in a rod of sufficient precision makes the most useful leveling rod.

The Tapes used may be of steel for verifying distances with some precision. For taking cross sections the Slope Tape is a most useful article. This is a tape having an unit of division equal to the unit of vertical measurement multiplied by the ratio of the slopes. In consequence, any horizontal measurement with this tape will be read off on it as the corresponding rise of the side slope of the cross section. If the distance E′ P, Fig 3 were measured with this tape, prepared for the slope of this cross section, the reading would be E E′ and not E′ P. The same would be true for any other slope with a tape made to fit it in a similar manner. Such a tape may be made by marking the plain back side of a metallic tape to a scale based on an unit equal to the unit of vertical measurement (commonly one foot) multiplied by the slope ratio. For the most common case where the slope ratio is one foot on one and one-half feet, use one and one-half feet for the unit of division on the back side of the tape. Transfer the divisions from the face of the tape by simple inspection, and number the unit marks. Use thick black paint with a stiff little brush for the marks and numbers.

0′.15	on	face	side	equals	0.1	on	slope,	or	back	side.
0′.30	"	"	"	"	0.2	"	"	"	"	"
0′.45	"	"	"	"	0.3	"	"	"	"	"
0′.60	"	"	"	"	0.4	"	"	"	"	"
0′.75	"	"	"	"	0.5	"	"	"	"	"

0'.90	on	face	side	equals	0.6	on	slope,	or back	side.
1'.05	"	"	"	"	0.7	"	"	"	"
1'.20	"	"	"	"	0.8	"	"	"	"
1'.35	"	"	"	"	0.9	"	"	"	"
1'.50	"	"	"	"	1.0	"	"	"	"
1'.65	"	"	"	"	1.1	"	"	"	"
&c.					&c.				

Observe the recurrence of the decimals on the face side and make the transfer mechanical. A slope tape for this particular slope ratio, and unit, is made and sold by the dealers in engineer's supplies. The zero of the two graduations should be at the same mark. This would be the case for a slope tape made as above directed. Attach to the slope tape a piece of metallic tape, somewhat longer than the half width of the widest road bed (or base) on the work, having the same zero mark as the slope tape, and with the numbers increasing in the opposite direction from this zero mark. The slope tape, or any metallic tape, should not be attached to any case, or reel, on this work, but be neatly coiled, putting a half twist in each coil so the tape will at once draw off without twist, and tied, when put away.

An Axe kept in excellent order should be used. Ordinarily at 3½ lbs. or 4 lbs. axe is suitable. It should be kept sharp enough to shave, and the head should be broad, square, and true. It should be "hung" true and kept so.

Twelve Wooden Pins, are needed on ordinary cross-section work. On rough, broken ground, more are necessary. These should be 18" or 20" long, about one inch in diameter, round, and pointed,—carefully, made of light strong wood. They are for marking places temporarily, on the cross sections.

Note Books containing any data needed on the work, particularly the one containing the Field Data, should be taken along. Another note book, to receive the records of the surveys of cross sections and other data should be provided. These last named books may be called the Field Data Book and the Cross Section Book.

Three, or more, No. 5, red hexagon, Faber, Lead Pencils, should be taken. When a single pencil gets lost it leaves the

leveler, or note keeper, in bad shape. The work should not be obliged to stop for want of a little foresight in carrying out pencils enough.

One or more good typewriter Erasers, of small size, will be needed.

For Marking Stakes, take three, or more, pieces of soft red keal. Soft lead pencils, and indelible pencils, are used. Lumber crayons are often of little service. Those made of grease and a coloring matter were formerly very serviceable but are about useless at present. The red keal may be obtained at drug, or grocery stores, or at general stores. A liberal supply should be kept on hand. Marks of good red keal have been found on a blazed tree, plainly legible, after thirty-four years.

A sportsman's "Skeleton Coat" is most useful for carrying the field equipment. It might be made water proof. A water proof sling pocket, or haversack, will do, or a light, short, over-all slicker.

The Field Data Book is used for the records of the levels for verifying the elevations of the pegs at all stations and plusses along the center line of the work, and from it are taken the data upon which the surveys of the cross sections are based. After the elevation of the level peg at a station, or plus, is verified, the correct elevation is transferred from the Field Data Book to its proper place in the Cross Section Book. The elevation of grade, and the rate of grade, for the same place on the line are also transferred. From time to time, as needed, the information noted on the vertical line beside the seventh column in the Field Data Book is transferred to a corresponding place in the Cross Section Book. The elevations of grade at plusses are worked out after those plusses have been measured, and entered in the Cross Section Book. By following this plan, the notes in the Cross Section Book may be kept in order, and properly spaced on the page, making a clear, plain record, easy to read.

The Cross Section Book is used for the records of the surveys of cross sections and all data relating to the earthwork, both as to volume, or measure, and construction. The accompanying form may be used on a great deal of work. Column 1 contains the elevations of the ground at all stations

and plusses as taken from the Field Data Book after being verified. Column 2 contains the elevations of grade as computed and verified, with the rates of grade, or changes of grade per station distance, entered along the line between columns 1 and 2, from station to station on the vertical curves in the grade line, and on straight grades between places designated by check marks. Column 3 contains the distances from the center stakes to the side stakes on the left hand side of the work, as the numbers increase, with the vertical distance of the ground at the side stake from the elevation of the roadbed, or center line grade elevation, written beneath, with its proper sign, + for a cut and — for a fill. Columns 4 and 5 contain information similar to that in 3 with respect to any important breaks in the surface of the ground between the center and side stakes on the left. Column 6 contains the station, ⊙ , and plus numbers, and beneath them the center cuts, +, or fills, —, found by subtracting the entries in 2 from those in 1. Columns 7, 8, and 9, for the right hand side of the work, are like 5, 4, and 3, for the left side.

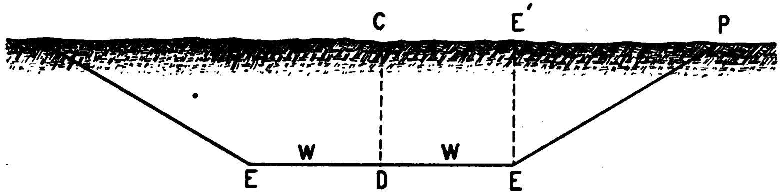


Fig. 3.

25. In Level Cross Sections, Fig. 3, the center height, C D, will be the same as the side heights, that is the vertical distance of C, and P, from the roadbed, or base, E E, will be the same. The distance out, C P, may be computed.

Let the ground between C and P be regarded as level.

Notation.

2 W = E E = width of roadbed, or base.

$$R = \frac{E'\,P}{E'\,E} = \text{ratio of slopes} = \text{base of side slope divided by}$$

its corresponding rise.

$+ H_C = C\ D =$ the cut or fill at the center stake $C =$ the center height, so called. $+$, indicates a cut, — a fill.

$D_P = C\ P =$ the distance out from the center stake C to the side stake P, — level measurement.

Use a slope tape.

Stakes at C and P should be blazed, or made smooth for marking, on both sides.

Mark the center cut or fill, taken from the Field Data Book, or the Cross Section Book, on the back side of the center stake at C, preceded by F. for a fill, or C. for a cut.

Measure out from C a distance W, at right angles to the line to the stake of the next higher number, either on a straight line or a curve. Estimate the right angle, ordinarily. Mark the place, E′, by one of the wooden pins.

Hold at E′ the mark, or place, on the slope tape that shows a slope reading equal to $C\ D = H_C$

Draw out the slope tape to its zero for the place of P.

Or, hold at C a distance mark on the piece of tape attached to the slope tape zero, that shows a reading equal to W, and draw out the slope tape to a slope reading equal to $C\ D = H_C$ for the place of P.

Or, find a slope reading on the slope tape equal to $C\ D = H_C$ and take from the distance side the corresponding horizontal distance; add W to this distance for the value of C P; and measure out this total distance for the place of P.

Or, compute the value of $C\ P = D_P = R\ H_C + W$, in case the slope tape is not at hand, and measure out C P to find the place of P.

Or find $\frac{W}{R}$. Add this to $C\ D = H_C$. Hold at slope reading at C equal to $H_C + \frac{W}{R}$. Draw out the slope tape to its zero for the place of P.

Mark the place of P with one of the wooden pins.

Check either method by one of the others.

Drive a stake at P, marked F., or C., as the case may be, together with $C\ D = H_C$, on one side, and the station num-

ber with the distance out C P = D_P, on the other. Face the letter F. or C. towards C.

Do the same on the other side of the center line. See if the two side stakes, P, and the center stake, C, are in line. If not, repeat the trial at squaring out, and correct the work.

Enter in the Cross Section Book in the L ± column and the R ± column, on the line of the station or plus number, the same entries that are on the side stakes, P, replacing C. with + and F. with —. Write the distance out with the cut or fill below it.

Hold the book so these entries in it will appear in the same relative position to those for the preceding cross section that the stakes bearing them do, on the ground, to the stakes of the preceding cross section, and draw, on the page, the surface lines between the two cross sections that best fit the ground; the lines in the book connecting the entries therein which are on the stakes on the ground between which said surface lines run. In case there is no choice between two surface lines draw both. There should be at least one surface line running to each point of survey on a coss section from the cross section each way from it. Surface lines from peg to peg along the center line are not drawn in the book because they are always in the same place. The same is true for the surface lines between side stakes, as cross sections should be taken so frequently that straight lines between these stakes will fit the ground the same as along the center lines. Considering all the surface lines, the surface of the ground between two successive cross sections must be divided into triangles, — likewise the page of the Cross Section Book.

26. In Cross Sections which are Not Level, Figs. 4 and 5, the vertical distance of the ground at C and P from the elevation of grade at D is not the same. The surface of the ground between C and P need not be straight, as indicated by the letters I.

The figures show four cases of setting side stakes. The same letter marks a similar point in each case. The method of finding the location of P, and of gathering the data for other places on the cross section, as at I, rests on the properties of similar triangles. A few points and lines are added to show

the proofs. Either case may be traced through by itself. These figures represent gently sloping ground where both side stakes can be set from one setting of the level. The ground may be so steep that the level must be set up more than once between C and P. The geometrical principles are the same for all cases.

Ground between C and P not level. Use a slope tape.

Notation.

The plus sign, +, refers to cuts, and the minus sign, —, to fills, except in a few equations, or when otherwise stated.

$2W = EE$ = width of road bed, or base.

R = ratio of slopes = base of side slope divided by its corresponding rise.

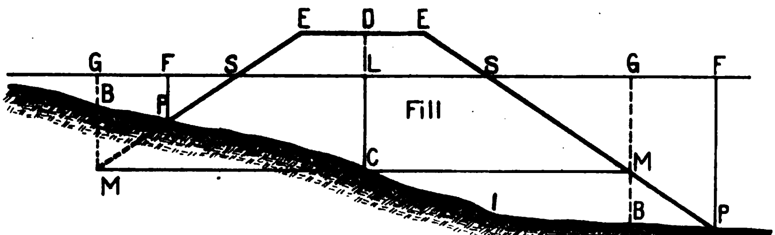


Fig. 4.

$\pm$ H = the cut, or fill, at any point on the cross section, that is the vertical distance of the surface of the ground from the grade elevation at D, with subscripts, when needed, to show what place it means as,

$\pm\ H_C = CD$ = the cut, or fill, at C.

The values of H are called "Heights" from the road bed E E, in both cuts and fills.

D = the level measured distance of the place where H is taken from the center stake, C, with the same subscripts, as,

D_P = the level measured distance of P from C.

$\pm\ b$ = the vertical distance at any designated place on a cross section between the surface of the ground and the slope line, with subscripts, when needed.

d = the distance from one point of survey on a cross section to the next in order, proceeding outwards from C to P, with subscripts, as d_1, d_2, d_3, &c.;

d_1 being between C and the first place I_1;
d_2 being between I_1 and I_2;
d_3 being between I_2 and I_3; and so on out to P.
$D_P = \Sigma\, d$ from C out to P.
L S is the line of sight of the level.
K = the rod reading on the ground at a place on a cross section, with subscripts, when needed.
K_C = L C = the rod reading on the level peg at C.
K_P = F P = the final rod reading at the place for P.
Similarly for other points.
T = the reading of the slope tape at the place of K, or H, or b, when held as directed, and with the same subscripts.
$b = T - K$.
$\pm$ Q = the reading on the distance side of the slope tape corresponding to any slope reading, T, with the same subscripts.

The ground may slope either upwards or downwards from C towards P, in either a cut or a fill.

Mark the center stake, C, with the cut or fill there, as directed for a level cross section.

Choose a favorable place for the level. Set it up with a line of sight L S.

Mark with a wooden pin a place B, on the cross section at the same distance from C that the side stake would be if the ground was level.

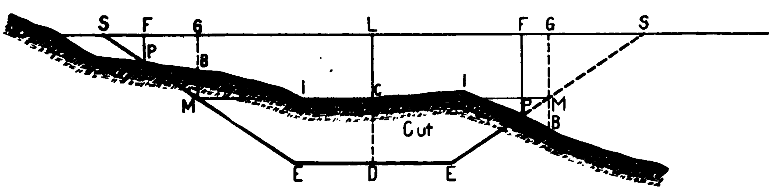


Fig. 5.

Take a rod reading, K_C = L C, on the center level peg at C.

Find on the slope tape a reading = K_C.

Hold the place at this reading on the slope tape to the pin which marks B on the ground.

For a cut draw out the slope tape level with its zero end **away from the center stake.**

For a fill draw out the slope tape level with its zero end **towards the center stake.**

By trial find P, that place on the ground where the rod reading $K_P = F\ P$ is the same as the reading there on the slope tape, $T_P = F\ P$, when it is held level with the right place on it kept at B. Work either way from B to find P, but keep the slope take drawn out as directed.

Mark P with a wooden pin.

Look on the distance side of the slope tape and see how far, in feet and tenths, P is from B. If P is beyond B, add this to the distance of B from C to get the distance of P from C. If P comes between B and C, subtract.

Measure from C to P directly and see that it is right.

Find the side height $\pm H_P = \pm H_C + (K_C - K_P)$, tnat is the vertical distance of the surface of the ground at P from the center grade elevation at D.

Check. See if $C\ P = D_P = R\ H_P + W$.

If not, review the work and correct the errors.

Enter in the Cross Section Book, at its proper place, this side height, with + before it for a cut, or — before it for a fill, and its distance out above it, in the R ± column, or the L ± column, according as it belongs to the right, or left, side of the work as the numbers increase along the center line,—see cross section notes.

Mark a stake for P with C. for a cut, or F. for a fill, and the side height H_C, on one side; and the station number with the distance out D_P on the other.

Drive this stake at P with the C. or F. facing C.

Notice if a line drawn from the ground at C to the ground at P will fit the surface well enough. If not, select places, I, at the changes in the surface between C and P so that a broken line from C passing through them in succession and thence to P, will fit well. Mark these places with the wooden pins as they are selected. These marks may usually be shifted a few tenths of a foot, if need be, so as to give values of d easy to multiply by, without making any difference in the volumes afterwards to be computed. In case such changes will make any material difference in the volumes the marks should not be shifted.

Measure d_1 from C to I_1; d_2 from I_1 to I_2; d_3 from I_2 to I_3; and so on out to P.

Extend the slope tape in the same manner as when finding P.

Take a rod reading K_1 and a slope tape reading T_1 at I_1, the first place out from C.

Do the same at I_2, I_3, &c., between C and P, at P, and at C also if found practicable. In the case of a fill where L S passes between C and D as in Fig. 4, C will be beyond the zero of the slope tape, and there may be intermediates between the zero of the slope tape and C. In such cases, draw out the slope tape to its zero, and mark the place with a wooden pin. Hold the zero at this pin. Reverse the slope tape. Draw it out level. Take K and T, as before. Give T a minus sign when the slope tape is reversed, and the results, $b = T - K$, will be correct.

Make a record similar to this.

	Sta.				
	C ±				
⊙	247			19.00	W = 10′.0 R = 1½
+	4.86			+ 6.0	H_P = 6′ 0
d	0.00	5.60	7.00	6.45	$\Sigma d = 19.00 = D_P$
T	15.10	11.40	6.74	2.45	
K	3.58	6.15	2.80	2.45	
b	+ 11.52	+ 5.25	+ 3.95	0.00	at P.

Check. The difference between two successive numbers on the line T, multiplied by R, equals the number on the line d standing above the smaller of the two.

Check. See if $\sum d = D_P$.

If these checks show errors, review the work and correct them.

Make this record item by item as the work goes on.

Drive no stakes at these intermediates, but the wooden pins should be left standing until the succeeding cross section is marked, surveyed, recorded, and the surface lines between the two last cross sections entered in the Cross Section Book.

Use the above form of record and prepare a suitable cross section book to receive it where there is much of such work to be done.

Where there are but one or two intermediates needed, occasionally, between the center and side stakes, the record can be made in an ordinary $4\frac{1}{2}'' \times 7\frac{1}{2}''$ field note book, as shown in the sample Cross Section Book. Either of two ways may be used for recording the intermediates, as follows:

Measure the distance, D, of each I from C. Subtract the K for each I from K_C and preserve the sign of the result. Add this result with its sign to H_C to get the H for the I at which K was taken. Record D and H in the columns marked I,—see sample Cross Section Book,—thus,

5.60	12.60	D
+ 2.3	+ 5.9	H

Or, add H_C and K_C, observing signs, (K_C is always +) and use the result with its sign as the Height of Instrument from the road bed, or grade elevation at D. Subtract from this H. I. any K, observing signs, as in leveling, and the result with its sign will be the value of the corresponding H for record as above.

The other way is to measure the distances, d, as above

directed and find each corresponding $b = T - K$, noting its sign. Record the d and b in the columns marked I, thus,

5.60	7.00	d
+ 5.3	+ 4.2	b

Set the side stake on the other side of the center line and get all the information needed on that side in the same manner as heretofore directed. Carefully record the position and data of every place.

Draw the surface lines neatly in the Cross Section Book, between the proper entries therein, as they will best fit the ground between the stakes and pins the entries represent. Where two surface lines fit equally well, draw both.

The first and last methods of recording intermediates are to be preferred, as the entries in the notes are the quantities used in computing the volume, except at C in the last form, where a constant must be added to H_C to get h_C for use in computing. The first method is the most preferable because the actual readings of the rod and tapes are entered in the record without any previous calculations.

27. In drawing out the **Slope Tape,** observe,

1. Keep it level.

2. Keep the proper mark at B.

3. For a cut draw the zero end away from C.

4. For a fill draw the zero end towards C.

28. In moving the **Rod** to find the place for P, observe,

1. If the rod reading, K, is larger than the tape reading, T, at any trial, move the rod to a place where the tape will read more.

2. If $K < T$ move the rod to a place where T will be smaller.

In other words it is only necessary to move the rod so as to bring the rod and tape readings together. No thought is necessary as to whether one is at work on a cut or a fill, whether the ground slopes one way or the other, whether the slope line is above or below the ground at any particular place, or as to any of the confusing perplexities of old time methods of cross sectioning. It may also be noticed that the place may be found on the ground for every stake and mark without any "figuring" whatever. Even the data for all the stakes and marks can be obtained and the record made without figuring. Figuring is required only when checks are applied or the marks to go on the side stakes found.

29. Additional Cross Sections may be needed, besides those provided for by the Field Data.

Be careful to take cross sections so frequently that straight lines will fit the ground between their center level pegs. If the levels as originally taken, do not provide for them, mark by wooden pins and level pegs the places for them. Set these level pegs on line between the station stakes, either on a straight line or a curve, and space them according to the Plus Table below. Enter in the record the plusses to these level pegs, measured from the station of the lower number. With the level and rod get the elevation of the level peg at any plus. Verify this elevation by rod readings on two or more level pegs whose elevations have been verified, or by reference to a checked B. M. Enter the correct elevation in the first column of the Cross Section Book. Compute the elevation of grade for the plus, as measured, and record it. Subtract the elevation of grade from the elevation of the level peg and record the result with its sign in the $C \pm$ column for H_C at the plus. Survey and record a cross section at the plus, as in any case. No stakes need be driven unless needed as a guide for the workmen. The places may all be marked temporarily with the wooden pins.

When the side stake, P, is set, or the place for it marked with a wooden pin, see if a straight line will fit the ground between it and the P on the same side of the preceding cross section. If not, take other cross sections between these two till straight lines will fit the ground between the side marks P. If the side line between two cross sections is straight on one side, but not on the other, it is only necessary to take additional cross sections on the side where the break is, as the volumes each side of the center line can be computed separately. Treat these cases the same as the cross sections at any other plusses.

30. **The Plusses** should all be measured, both the old and the new, from the station of the lower number preceding them. If the level pegs for the old plusses cannot be found, set new ones, and get and verify their elevations. The pegs at the old plusses may not be at the most convenient distances from their preceding stations. New pegs should be set according to the Plus Table, below, and their elevations found and verified for use in cross sectioning. Destroy the old pegs in such cases.

The places for all plusses should be chosen according to the Plus Table, below. But little shifting of the old plusses will be required to conform to this table. New ones can be located by it. The ground, or the circumstances of the work, would be extremely unusual where this table could not be used. The entries in it advance by 0'.27, thus providing for any requirement likely to be met with. Use this table in fixing the places of all plusses where the station distance is 100 feet. The prismoids of the formation will then have their lengths convenient for the calculation of their volumes, as 33.75, 51.84, 85.59, instead of 34, 52, and 85.5. This is because earth work is commonly measured in feet and paid for by the cubic yard. Multiplying by 34 and dividing by 27,—or the equivalent,—is a more serious matter than multiplying by $1\frac{1}{4} = \frac{33.75}{2.7}$, especially when repeated scores and hundreds of times. The closing length of the work between two stations

100 feet apart will always exceed the value of one of these plusses by 0.10 of a foot. The difference between any two plusses in the table as well as the sum of any number of them, and any decimal multiple of any of them, will give a number of like properties. Consequently this table can be used to space cross sections on any kind of work whatever the station distance may be, or where there are no regularly spaced stations.

These plusses are selected for this table so that instead of having to multiply by them and divide by 27 in the final part of the volume calculations, it will be necessary only to multiply by the number given in the "Mult." column and subtract from this product that multiple of the multiplicand that is named in the "Sub." column, or perform such other simple work as the table indicates. The factors given in the table are to be used as factors with the multiplicand, in all cases, never with the product by a previous factor.

PLUS TABLE.

Plusses.	Mult.	Sub.	Remarks.
0.27	.01		
0.54	.02		
0.81	.03		
1.08	.04		
1.35	.05		
1.62	.06		
1.89	.07		
2.16	.08		
2.43	.09		
2.70	.10		
2.97	.11		
3.24	.12		
3.51	.13		
3.78	.14		
4.05	.15		
4.32	.16		
4.59	.17		
4.86	.18		
5.13	.19		
5.40	.20		
5.67	.21		
5.94	.22		
6.21	.23		
6.48	.24		
6.75	.25		
7.02	.26		
7.29	.30	.03	
7.56	.30	.02	
7.83	.30	.01	
8.10	.30		
8.37	.31		
8.64	.32		
8.91	.33		

Plusses.	Mult.	Sub.	Remarks.
9.18	.34		
9.45	.35		
9.72	.40	.04	
9.99	.40	.03	
10.26	.40	.02	
10.53	.40	.01	
10.80	.40		
11.07	.41		
11.34	.42		
11.61	.43		
11.88	.44		
12.15	.50	.05	
12.42	.50	.04	
12.69	.50	.03	
12.96	.50	.02	
13.23	.50	.01	
13.50	.50		
13.77	.51		
14.04	.52		
14.31	.53		
14.58	.60	.06	
14.85	.55		
15.12	.60	.04	
15.39	.60	.03	
15.66	.60	.02	
15.93	.60	.01	
16.20	.60		
16.47	.61		
16.74	.62		
17.01	.70	.07	
17.28	.64		
17.55	.65		
17.82	.66		
18.09	.70	.03	
18.36	.70	.02	
18.63	.70	.01	
18.90	.70		
19.17	.71		

Plusses.	Mult.	Sub.	Remarks.
19.44	.80	.08	
19.71	.73		
19.98	.80	.06	
20.25	1.00	.25	
20.52	.80	.04	
20.79	.77		
21.06	.80	.02	
21.33	.80	.01	
21.60	.80		
21.87	.81		
22.14	.82		
22.41	.83		
22.68	.84		
22.95	1.00	.15	
23.22	1.00	.14	
23.49	.90	.03	
23.76	.88		
24.03	.90	.01	
24.30	.90		
24.57	.91		
24.84	.92		
25.11	.93		
25.38	1.00	.06	
25.65	1.00	.05	
25.92	1.00	.04	
26.19	1.00	.03	
26.46	1.00	.02	
26.73	1.00	.01	
27.00	1.00		
27.27	1.01		
27.54	1.02		
27.81	1.03		
28.08	1.04		
28.35	1.05		
28.62	1.06		
28.89	1.07		
29.16	1.20	.12	
29.43	1.10	.01	

Plusses.	Mult.	Sub.	Remarks.
29.70	1.10		
29.97	1.11		
30.24	1.12		
30.51	1.13		
30.78	1.14		
31.05	1.15		
31.32	1.20	.04	
31.59	1.30	.13	
31.86	1.20	.02	
32.13	1.20	.01	
32.40	1.20		
32.67	1.21		
32.94	1.22		
33.21	1.23		
33.48	1.24		
33.75	1.25		Or, divide by 0.80.
34.02	1.40	.14	
34.29	1.30	.03	
34.56	1.30	.02	
34.83	1.3	.01	
35.10	1.3		
35.37	1.31		
35.64	1.32		
35.91	1.33		
36.18	1.34		
36.45	1.5	.15	
36.72	1.4	.04	
36.99	1.4	.03	
37.26	1.4	.02	
37.53	1.4	.01	
37.80	1.4		
38.07	1.41		
38.34	1.42		
38.61	1.43		Or, mult. 1.3 and add 0.13 of the number.
38.88	1.44		
39.15	1.5	.05	
39.42	1.5	.04	

Plusses.	Mult.	Sub.	Remarks.
39.69	1.5	.03	
39.96	1.5	.02	
40.23	1.5	.01	
40.50	1.5		
40.77	1.51		
41.04	1.52		
41.31	1.53		
41.58	1.54		Or, mult. 1.4 and add 0.14 of the number.
41.85	1.55		
42.12	1.3		Add 0.26 of the number.
42.39	1.6	.03	
42.66	1.6	.02	
42.93	1.6	.01	
43.20	1.6		
43.47	1.61		
43.74	1.62		Or, mult. 1.8 and sub. 18.
44.01	1.63		
44.28	1.64		
44.55	1.5		Add 0.15 of the number.
44.82	1.66		
45.09	2.0	.33	
45.36	2.0	.32	
45.63	2.0	.31	
45.90	2.0	.30	
46.17	1.71		
46.44	1.72		
46.71	1.73		
46.98	1.8	.06	
47.25	2.0	.25	
47.52	1.6		Add 0.16.
47.79	1.8	.03	
48.06	2.0	.22	
48.33	2.0	.21	
48.60	2.0	.20	
48.87	1.81		
49.14	1.82		
49.41	1.83		

Plusses.	Mult.	Sub.	Remarks.
49.68	1.84		
49.95	1.85		
50.22	1.86		
50.49	2.0	.13	
50.76	2.0	.12	
51.03	2.0	.11	
51.30	2.0	.10	
51.57	2.0	.10	Add .01
51.84	2.0	.08	
52.11	2.0	.07	
52.38	2.0	.06	
52.65	2.0	.05	
52.92	2.0	.04	
53.19	2.0	.03	
53.46	2.0	.02	
53.73	2.0	.01	
54.00	2.0		
54.27	2.01		
54.54	2.02		
54.81	2.03		
55.08	2.04		
55.35	2.05		
55.62	2.06		
55.89	2.1	.03	
56.16	2.1	.02	
56.43	2.1	.01	
56.70	2.1		
56.97	2.11		
57.24	2.12		
57.51	2.13		
57.78	2.14		
58.05	2.15		
58.32	2.16		
58.59	2.2	.03	
58.86	2.2	.02	
59.13	2.2	.01	
59.40	2.2		
59.67	2.21		

Plusses.	Mult.	Sub.	Remarks.
59.94	2.22		
60.21	2.23		
60.48	2.24		
60.75	2.25		
61.02	2.26		
61.29	2.3	.03	
61.56	2.3	.02	
61.83	2.3	.01	
62.10	2.3		
62.37	2.31		
62.64	2.32		
62.91	2.33		
63.18	2.34		
63.45	2.35		
63.72	2.4	.04	
63.99	2.4	.03	
64.26	2.4	.02	
64.53	2.4	.01	
64.80	2.4		
65.07	2.41		
65.34	2.42		
65.61	2.43		
65.88	2.44		
66.15	2.5	.05	
66.42	2.5	.04	
66.69	2.5	.03	
66.96	2.5	.02	
67.23	2.5	.01	
67.50	2.5		
67.77	2.51		
68.04	2.52		
68.31	2.53		
68.58	2.54		
68.85	2.55		
69.12	2.56		
69.39	2.6	.03	
69.66	2.6	.02	
69.93	2.6	.01	

Plusses.	Mult.	Sub.	Remarks.
70.20	2.6		
70.47	2.61		
70.74	2.62		
71.01	2.63		
71.28	2.64		
71.55	2.65		Or, div. 2.0 and mult. 5.3.
71.82	2.66		
72.09	3.0	.33	
72.36	3.0	.32	
72.63	3.0	.31	
72.90	3.0	.30	
73.17	3.0	.30	Add .01
73.44	3.0	.30	" .02
73.71	3.0	.30	" .03
73.98	3.0	.30	" .04
74.25	3.0	.25	
74.52	3.0	.24	
74.79	3.0	.23	
75.06	3.0	.22	
75.33	3.0	.21	
75.60	3.0	.20	
75.87	3.0	.19	
76.14	3.0	.18	
76.41	3.0	.17	
76.68	3.0	.16	
76.95	3.0	.15	
77.22	3.0	.14	
77.49	3.0	.13	
77.76	3.0	.12	
78.03	3.0	.11	
78.30	3.0	.10	
78.57	3.0	.09	
78.84	3.0	.08	
79.11	3.0	.07	
79.38	3.0	.06	
79.65	3.0	.05	
79.92	3.0	.04	
80.19	3.0	.03	

Plusses.	Mult.	Sub.	Remarks.
80.46	3.0	.02	
80.73	3.0	.01	
81.00	3.0		
81.27	3.01		
81.54	3.02		
81.81	3.03		
82.08	3.04		
82.35	3.05		
82.62	3.06		
82.89	3.07		
83.16	3.08		
83.43	3.09		
83.70	3.1		
83.97	3.11		
84.24	3.12		
84.51	3.13		
84.78	3.14		
85.05	3.15		
85.32	3.16		
85.59	3.2	.03	
85.86	3.2	.02	
86.13	3.2	.01	
86.40	3.2		
86.67	3.21		
86.94	3.22		
87.21	3.23		
87.48	3.24		
87.75	3.25		
88.02	3.3	.04	
88.29	3.3	.03	
88.56	3.3	.02	
88.83	3.3	.01	
89.10	3.3		
89.37	3.31		
89.64	3.32		
89.91	3.33		
90.18	3.34		
90.45	3.35		

Plusses.	Mult.	Sub.	Remarks.
90.72	3.36		
90.99	3.4	.03	
91.26	3.4	.02	
91.53	3.4	.01	
91.80	3.4		
92.07	3.41		
92.34	3.42		
92.61	3.43		
92.88	3.44		
93.15	3.45		
93.42	3.46		
93.69	3.5	.03	
93.96	3.5	.02	
94.23	3.5	.01	
94.50	3.5		
94.77	3.51		
95.04	3.52		
95.31	3.53		
95.58	3.54		
95.85	3.55		
96.12	3.56		
96.39	3.6	.03	
96.66	3.6	.02	
96.93	3.6	.01	
97.20	3.6		
97.47	3.61		
97.74	3.62		
98.01	3.63		
98.28	3.64		
98.55	3.65		
98.82	3.66		
99.09	3.7	.03	
99.36	3.7	.02	
99.63	3.7	.01	
99.90	3.7		
100.17	3.71		

31. **When the Level must be set more than once** between C and P, find and mark B, as in any case, and hold the slope tape as usual. Fig. 6.

Keep a memorandum of the rod readings $\pm$ K, from $+ K_C$ to $- K_P$, and the distance readings on the slope tape, from $+ Q_B$, corresponding to $+ K_C$, to $- Q_P$, corresponding to $- K_P$, each in the form of peg level notes, thus,

$+K_C$		$+Q_C$	
$+K_1$	$-K_1$	$+Q_1$	$-Q_1$
$+K_2$	$-K_2$	$+Q_2$	$-Q_2$
$+K_3$	$-K_3$	$+Q_3$	$-Q_3$
&c. to	&c. to	&c. to	&c. to
$+K_n$	$-K_n$	$+Q_n$	$-Q_n$
	$-K_P$		$-Q_P$

Record $+ K_C$ and $+ Q_B$.

Take, and record, a rod reading, $- K_1$, on the first peg, C_1, set wherever desirable for leveling from C out to P.

Draw out the tape to a slope reading equal to $- K_1$ and mark this place on the ground, B_1 in the figure.

Record $- Q_1$ corresponding to $- K_1$.

Move the level. Set it up wherever desirable for leveling out to P, with a line of sight $L_1 S_1$.

Take and record a rod reading $+ K_1$ on C_1.

Hold at B_1 a slope reading equal to $+ K_1$.

Record $+ Q_1$ corresponding to $+ K_1$.

Take, and record, a rod reading, $- K_2$, on C_2, the second peg out from C.

Draw out the tape to a slope reading equal to $- K_2$, and mark this place on the ground, B_2 in the figure.

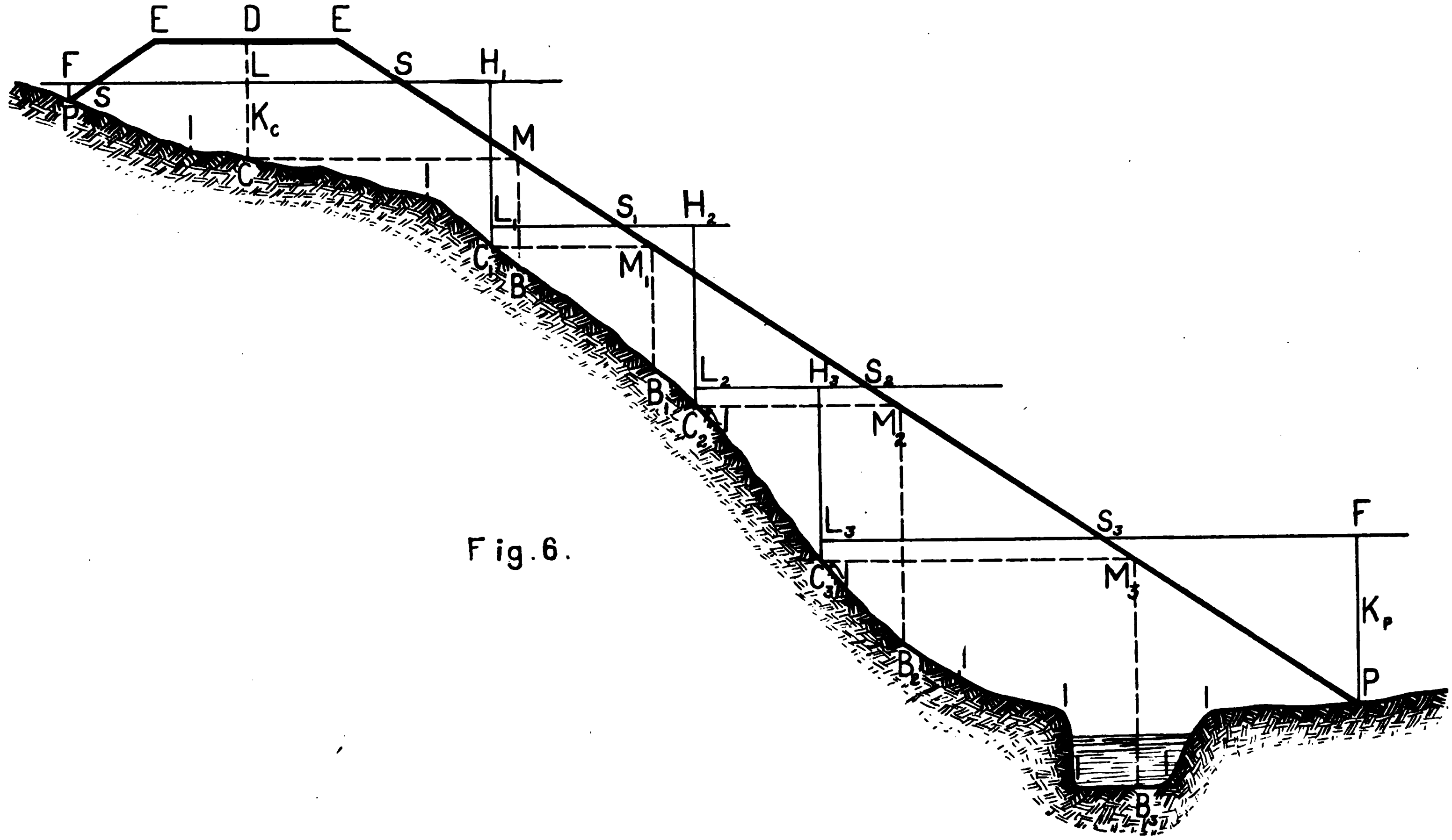

E
D
E
F
L
S
H_1
S
P
K_c
M
C
L_1
S_1
H_2
C_1
B
M_1
L_2
H_3
S_2
B_1
C_2
M_2
L_3
S_3
F
C_3
M_3
K_p
B_2
P
B_3

Fig. 6.

Record — Q_2 corresponding to — K_2.

Move the level. Set it up with a line of sight $L_2 S_2$.

Take, and record, a rod reading, + K_2, on C_2.

Hold at B_2 a slope reading equal to + K_2.

Record + Q_2 corresponding to + K_2.

Take, and record a rod reading, — K_3, on C_3, the third peg out from C.

Draw out the tape to a slope reading equal to — K_3, and mark this place on the ground, B_3 in the figure.

Record — Q_3 corresponding to — K_3.

Move the level. Set it up with a line of sight $L_3 S_3$.

Take, and record, a rod reading, + K_3, on C_3.

Hold at B_3 a slope reading equal to + K_3.

Record + Q_3 corresponding to + K_3.

Proceed in the manner above outlined till P can be reached which may be known by comparing the slope reading from the tape, with the rod reading, at places on the ground.

Find the place where the slope reading and the rod reading on the ground are the same. This will be the place for P, as in other cases.

Record — K_P, as a minus sight.

Record — Q_P, corresponding to — K_P.

Find $H_P = H_C + \Sigma$ (K, from + K_C to — K_P).

$$\begin{aligned} D_P &= D_B + R\,\Sigma\,(K, \text{ from } + K_C \text{ to } - K_P). \\ &= D_B + \Sigma\,(Q, \text{ from } + Q_B \text{ to } - Q_P). \end{aligned}$$

Check. $D_P = R\ H_P + W$. The distance from C out to P may be measured directly.

Having verified the place for P mark and drive a stake there.

Notice when leveling out from C to P, where intermediates are needed, as at the places marked I, in the figure. Mark these places with the wooden pins, set by distance measurement, and so spaced as to give convenient numbers when computing. Leave these pins standing till the next cross section has been surveyed and the surface lines to it recorded.

Take a rod reading, — K, a slope reading, T, and a distance reading, — Q, at each of these places whenever most convenient.

It is better to record the *d*, K, and T, for each I, instead of the *d* (or D) and $b = T - K$.

Make a temporary record of — Q at each I.

Check. Between any two places, $\Sigma\, d = \Sigma\, Q$, including + Q at the place nearer C, and — Q at the place farther from C. This applies between C and P.

Other checks will appear after some experience.

Record all surface lines to each point of survey on the cross section.

32. Sometimes the center line of the work may run along the face of a steep hill side in such a way that the side stakes for a considerable distance along the line will all come very much above, or below, the surface of the ground at the center stakes, C. In such cases it is convenient to pass from one cross section to the next without returning to the center line to begin the survey of each cross section. It may also be found desirable to carry on the survey of two, or more, cross sections at the various settings of the level. The methods are outlined below.

33. To Stake Out One Cross Section from Another, measure out from the center stake, C, of the new cross section the same distance the tape mark, B_n, in use on the cross section for which the level is set, is from its center stake, and mark the place on the ground. Hold here the same slope tape reading that was in use there, either — K or + K, as the case may be. It will be — K provided a new mark B_n, which has been set preparatory to moving the level is used for reference, otherwise it will be + K. It may be noted that the distance of any tape mark, B, from its center stake, C, equals $D_B + \Sigma$(Q, from + Q_B to — Q for B_n referred to), or equals Σ (Q, from + Q_C to — Q for B_n referred to), or may be measured directly from C. Measure the distance directly from C and compare this measurement with the same distance obtained in one of the other ways to verify it before making it the basis for beginning, or continuing, the survey of another cross section.

Find the change in the elevation of grade between the centers, D, of the two cross sections.

For a level grade, continue the work with the tape as set.

For a rising grade from the first to the second cross section, draw out the tape to a slope reading equal to this rise in grade and mark the place on the ground. Hold here the zero of the slope tape and go on as usual.

For a falling grade from the first to the second cross section, draw out the slope tape to its zero, and mark the place on the ground. Hold here a slope reading equal to the fall in grade between the two cross sections, and go on as usual.

If the level must be moved to reach the side stake, P, proceed as directed when leveling out on a cross section.

If the level must be moved to carry the work towards the center stake, C, of the second cross section, proceed as follows, —the same as when leveling out:

Take, and record, a rod reading, — K, on a turning point, C_n, convenient for leveling towards the center stake, C.

Draw out the tape to a slope reading, equal to — K, and mark this place on the ground.

Record — Q corresponding to — K.

Move the level. Set it up in a desirable place.

Take, and record, a rod reading, + K, on the turning point.

Hold at the tape mark on the ground a slope reading, + K.

Record + Q corresponding to + K.

Go on as before. Continue the process at will.

Find Σ K for the difference in elevation between any two places.

Find Σ Q for the distance between any two places.

Take T, and K, and Q, for any needed intermediates at any convenient time and make the necessary records.

Continue the work to the center stake, C, if desirable, and check the levels there, getting and recording all needed information on the way.

Where the cross sections are near each other, carry out two or more, as the level is moved on, either by having a set of men for each cross section, or by shifting the men. Keep the records of *d*, K, T, and Q, for each cross section, entirely separate.

These operations may be repeated for any number of cross sections, and all intermediates that are in reach also determined, and recorded.

At last peg back to the center line and check the levels there.

Keep the distance measurements well checked by measuring out from the center stake on each cross section.

Complete the notes of these cross sections by additional work, where needed.

Record all surface lines, and notice particularly the extreme outside lines, to see that they fit the ground.

On rough ground, keep a record, it may be temporary, of the level pegs, and number them, when working on any cross section, for use in checking the levels, or in taking them up.

Experience will reveal checks on such work, in great numbers and variety.

34. **On Cross Sections which are Part Cut and Part Fill,** Figs. 7 and 8, such as are met with on some hill sides, proceed as follows:

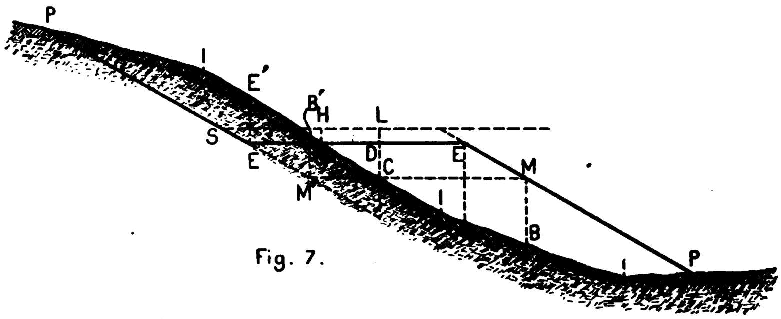


Fig. 7.

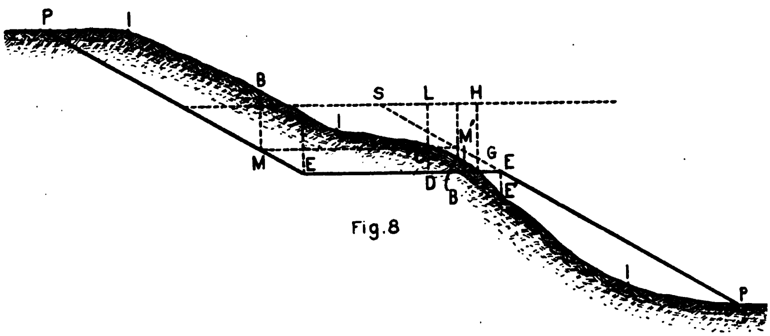


Fig. 8

Mark the center stake.

Measure out W for the side which is different from the center, C, either cut or fill, and mark E′ on the ground.

Hold at E′ the zero of the slope tape.

Draw the tape **towards the center stake,** C, to a slope reading equal to the center cut or fill, $\pm H_C$, and Mark B′ on the ground even with this reading.

Take a rod reading, $+ K_C$, on the level peg at C.

Hold at B′ a slope reading on the tape equal to $+ K_C$.

When there is a cut on the side of B′, draw out the slope tape with its zero end **away from** C.

When there is a fill on the side of B′, draw out the slope tape with its zero end **towards** C.

Then proceed as in any case.

To find G, the grade point, take a slope reading, T_E', at E′.

By trial, find, G, that place on the ground where the rod reading, $K_G = H\ G = T_E' = + K_C \pm H_C$.

Check the place, and the rod reading by using both values.

Treat G as an intermediate. Measure its distance from the last intermediate, I, or from C, as may be. Record the data in the proper place in the Cross Section Book.

Draw the surface lines to G as to any other place. Surface lines must connect grade points entirely across the whole work.

When taking the slope reading, T_E', at E′, take a rod reading, $- K_E'$, there, and record them both for use in computing the volumes of the prismoids, or the area of the cross section.

If it is desirable to get G at the first setting of the level, and there is a cut at C, do not set the level so high that $+ K_C + H_C$ will be greater than the rod will read. In case there is a fill at C, set the level so $+ K_C$ will be numerically greater than $- H_C$ by a half a foot, say.

Continue the work on out to P, on the side G is on.

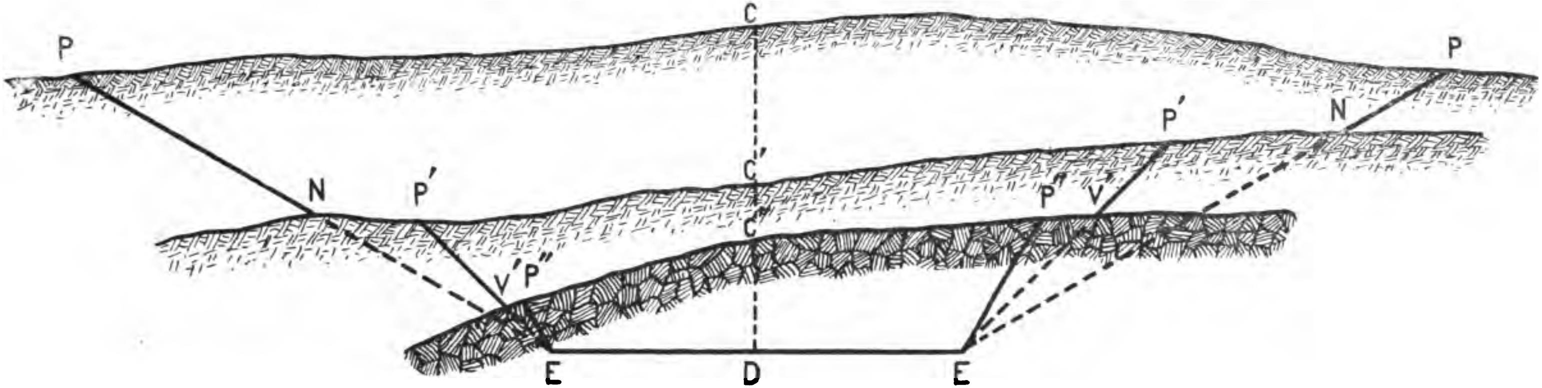
On the side which is the same as the center, C, either cut or fill, proceed as though the cross section was all cut, or all fill, as the case may be.

Get, and record, the data for all needed intermediates, as at I in the figures, whenever convenient, only take those on the cut side of G while the tape is set for a cut, and on the fill side of G when the tape is set for a fill.

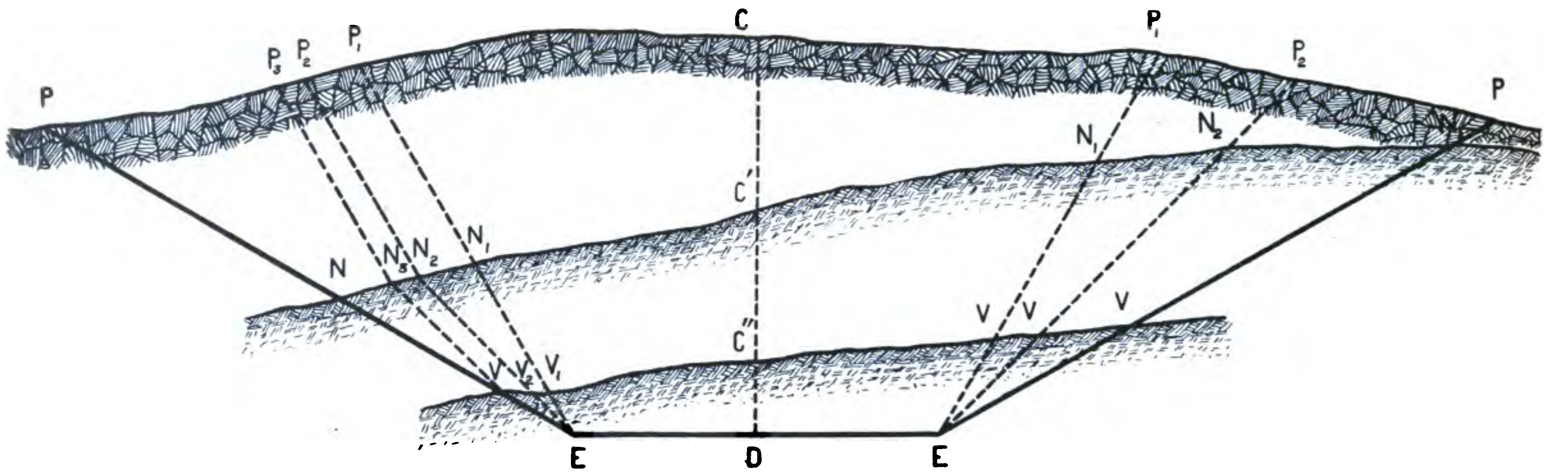
Draw all the surface lines, as usual.

35. **Compound Cross Sections.** Figs. 9 and 10 come in cuts where more than one kind of material is met with. With the softer materials uppermost the conditions are favorable. With the reverse great difficulties are likely to be met. There is no telling what to expect without borings are made, unless surrounding objects give some clue. The materials may occur in any order, form, or condition. Where changes are expected that will endanger the work, make borings till the locations of the different materials are learned. This being done, very likely the cross sections can be planned before hand.

When the Soft Materials are Uppermost, cross section as if all the materials were like those in sight. In Fig. 9 one would get the side stakes P at first. When the surface of the next material is reached, uncover it out to N, and cross section it, determining and recording the data for N, P′, and C′, with all intermediates and surface lines. Leave the spaces N P′ as berms to keep the soft material from washing into the cut. These may even have to be widened, giving more width



Soft Materials on Top.
Fig. 9.



Hard Materials on Top.
Fig.10.

to the cut on top of ground. The prismoid outside of P N will be of a well known form and as easily figured as any. When the surface of the next material is reached, uncover it out to V′, and cross section it determining and recording the data for V′, P″, and C″ with all needed intermediates and surface lines. The wash, or drainage, may require a still further widening of the whole upper part of the cut. Fig. 9 shows a theoretical case.

When the Hard Materials are Uppermost, as in Fig. 10, the stakes might be set at P_1, then as at P_2 on the right, and finally at P. But even this slope will likely prove too steep and give trouble, requiring a still further widening of the cut, or special works. If water did not make trouble, theoretically, the lines would go as shown on the left hand side of Fig. 10, first P_1 N_1, then P_2 N_2 V_2, then P_3 N_3 V E.

None of these cross sections present any especial difficulty. It is only necessary to do the cross sectioning before the lower materials are disturbed and to have those above removed so as to get at the places N and V in season. Be careful to get data for all intermediates, and record the same,—also all surface lines. It is well to anticipate the requirements in such cases, by extending the cross profile beyond the sidestakes, P, when the cross sections are first taken, determining points that will be intermediates, and all surface lines to them.

36. **Grade Stakes** are set to mark the points G in Fig. 1 where the grade line A B C D E F intersects the surface line on the profile. These stakes come on the center line of the work, and are shown in plan in Figs. 11, and 12. The road bed, or base, being a plane of a width 2W will cut the surface of the ground in a straight, or broken line. This fact makes it necessary to find the position of two or more points in order to get all the dimensions of a cut or a fill on lines of communication, such as highways, streets, and railways. W for a cut is larger than W for a fill on such work. Five points are therefore required to determine the line of intersection of the road bed with the surface of the ground. More may be needed, but at least five will be required. If

the center line crosses a hill side squarely, or in the direction of steepest slope, all five points may come in a straight line square across the center line, as shown in Fig. 11, a plan of a square entrance, meaning that the road bed enters, or leaves, the ground on a line square across the center line. When the plane of the road bed cuts the hill side obliquely there will be an oblique entrance shown in Fig. 12. This line of intersection G G G G G, Figs. 11 and 12, is the line where the cut and fill join, and marks the end of each.

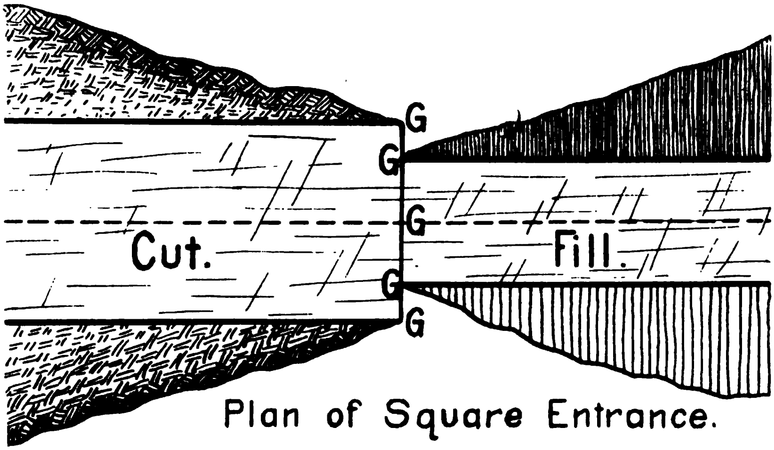


Fig. 11.

These are the places indicated in the Cross Section Notes opposite p. 64, where the cuts and fills are zero, as at

10.0	6.0	95.8	3.6	6.0	10.0
0.0	0.0	0.0	0.0	0.0	0.0

In the case of a square entrance, Fig. 11, only the center line grade point need be marked and the notes will appear thus,

10.0	6.0	40.5	6.0	10.0
0.0	0.0	0.0	0.0	0.0

with surface lines to these entries as in any other case. The grade points are either side stakes or intermediates, and are recorded as such, with surface lines as usual. Grade points should be connected by surface lines entirely across the whole work.

Plan of Oblique Entrance

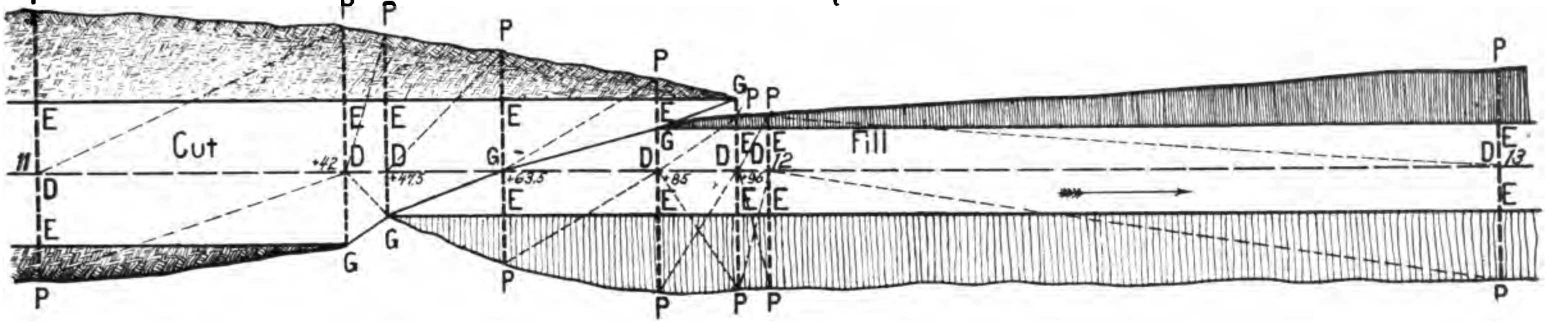


Fig. 12.

In Fig. 12 appear the five principal points standing on any kind of a broken line G G G G G. For the workmen only the center line grade point usually need be marked by a stake. All the others may be marked by wooden pins, temporarily. Mark them all. See if straight lines, joining them in succession, fit the ground well. If not, other grade points must be found and marked at such distances from the center line that these, with those first marked, will define a broken line that will fit the ground well.

The section P P, at (⊙) 11, (⊙) 12 and (⊙) 13 are regular cross sections at stations. The plusses for the various grade points are given and cross sections shown at each. The surface lines are the diagonal dashed lines and the lines from G to G. Each G in Fig. 12 corresponds to G in Figs. 7 and 8, and is a grade point on the cross section through it. For G at + 63.5, and at + 85, there is a cut on the left and a fill on the right, as the arrow points, and the station numbers increase. So may each G be studied.

Always take a cross section at a grade point.

When G is at the side of the road bed find the center height abreast of it and complete the cross section with all intermediates and surface lines. Sometimes this has not been done. Needed information was lacking which could never be supplied, as the workmen had destroyed the ground surface, or covered it up. Better take full notes, numerous cross sections, record everything, and by no means ever think of trusting anything to memory.

37. **For Setting Grade Stakes,** Fig. 13, set up the level in a convenient place with a line of sight L H, so the rod can be read on the level peg at the center stake, C, where the cut, as at (⊙) 11, is C D = $+ H_C$, or the fill, as at (⊙) 12, is C D = $- H_C$.

When working from the cut side, waste no elevation by setting the level too high. When working from the fill side, set the level as high as practicable if there is still higher ground to be reached.

Take a rod reading, $+ K_C$, on C.

Find $+ K_C \pm H_C$ (+ for a cut, — for a fill, before H_C.)

For a level grade, Fig. 13, by trial along the center line, find G, where the rod reading is $H\ G = L\ D = +\ K_C + H_C$.

Drive a stout plug at G till the rod reads H G when held on it.

Measure the distance of G from the preceding station, as the numbers increase.

Drive a large, broad, flat, witness stake beside the plug, marked G R A D E on one side, and the station number and plus on the other. Face the word towards the preceding station.

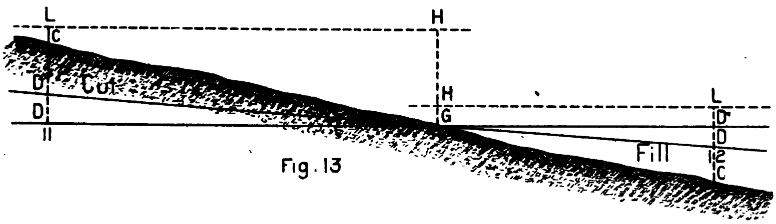


Fig. 13

Record the plus distance in the Sta. column in the Cross Section Book, with 0.00 under it.

Take and record, a cross section through G, with all the surface lines.

Set up two stakes at a distance W from the center line, on the right, to range by, and find G by trial, just as before, on the right hand side line of the road bed.

Drive no stake here but mark the place with a wooden pin.

Find, and mark with a wooden pin, the plus point on the center line abreast of G on the side line. Estimate the right angle as in cross sectioning.

Measure the plus distance from the preceding station.

Drive a level peg at the plus point.

Record the plus in the Sta. column.

Record the grade point in the R ± column using W with 0.0 under it.

Take a rod reading, — K'_C on the level peg at the plus point.

Find the elevation of the level peg.

Verify the elevation by other rod readings on other level pegs whose elevations are on record till satisfied that it is correct.

Record this elevation in the El. Gd. column.

Subtract the elevation of grade from this recorded elevation, and enter the result under the plus record in the Sta. column for the $\pm$ H'_C at the plus point.

Use this $\pm$ H'_C for the center height, and K'_C (with a $+$ sign now) as the center rod reading, at the plus point, and survey, and record a cross section there, upon which G on the right side line of the road bed will be a grade point.

Leave wooden pins standing to mark all the points of survey on this cross section.

In the same manner find the other three grade points, if they are all needed; locate their plus points on the center line; measure the plusses; record them; take, and record, cross sections at each plus point; and leave wooden pins to mark all the various points of survey.

Examine the ground and see if straight lines joining the various grade points in succession across the work will fit the ground well enough.

If not locate other grade points at such distances from the center line that the broken line joining the grade points in succession will fit the ground.

Determine the positions of these additional grade points and take, and record, cross sections through each, as above directed for the side line grade points.

With the wooden pins still standing at all these various points of survey record all the surface lines joining them.

Notice if there is at least one surface line running to each point of survey on every cross section from the cross section each way from it, as recorded in the Cross Section Book, and also be sure the page is divided into triangles by the lines between the records of the cross sections.

For an inclined grade, Fig. 13, find G, on the proper line, where the rod reading, $K_G = H\,G = + K_C \pm H_C - kl = L\,C \pm C\,D - k \times G\,D$, where k is the rise or fall of the grade per foot ($+$ up, $-$ down), and $l = G\,D$, the distance in feet of G from the station where the center rod reading, K_C, was taken.

At the first trial select a place as near as can be estimated to the right place for G.

Mark this place with a wooden pin.

Measure its distance in feet from C where $+ K_C$ was taken.

Multiply this distance by k.

Subtract the product with its sign from $+ K_C \pm H_C$.

See if the rod reads this result when held up on the place selected.

If not, try again.

Leave the wooden pin standing that marks the place first selected.

Select a new place.

Measure its distance in feet from the wooden pin.

Multiply this distance by k.

Change the former rod reading by this amount, noticing particularly whether the movement has been made up grade or down.

Repeat these trials till G appears to have been found.

Mark the place with a wooden pin.

Measure its plus, l.

Find $+ K_C \pm H_C - k\,l$.

See if this is the rod reading at the last place selected.

If not, make further trials.

Verify the place also by reference to the level peg at the station beyond.

When satisfied the right place for G has been found pull up all the wooden pins used to locate it, except the one which marks it.

On the center line set a plug and a witness stake as directed on a level grade.

Make a record of the work as completed.

Take, and record, a cross section through the center line grade point.

After the work is finished for the center line grade point, the same process is followed to find the stakes to find the grade points on the side lines of the road bed, using stakes to range by, as described for a level grade. Find, and verify, the elevations of the level pegs on the center line at the plus points abreast of the grade points on the side lines of the road bed.

Compute the elevations of grade for them. Find the center heights. Complete a cross section through each grade point, and also its record.

If additional grade points are needed find them in the same way as those on the side lines of the road bed were found, and take, and record, cross sections through each.

38. **Three Sticks** may be used to set Grade Stakes with, Fig. 14.

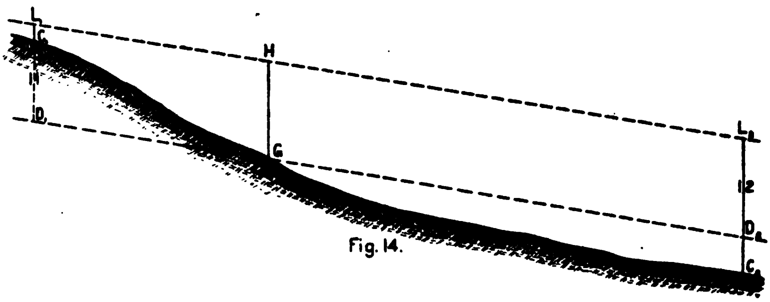


Fig. 14.

Measure off from the squared top of a stick, any distance, $C_1 L_1$, to clear stuff on the ground. Cut a notch to mark this distance. Sharpen the stick below the notch.

Drive it at C_1, plumb, till the notch is at the top of the level peg there.

Set up a stick, or pole, at C_2, in the same way, so its top, L_2, will stand above the level peg there, at a height $C_2 L_2 = C_1 L_1 + C_1 D_1 + C_2 D_2 = C_1 L_1$ plus the cut at C_1 and the fill at C_2.

Cut a third stick with both ends squared, and of a length $L_2 D_2 = L_1 C_1 + C_1 D_1$

Sight from L_1 to L_2.

By trial along the center line, find G on the ground where the third stick will just reach vertically from it up to the line $L_1 L_2$.

Complete the work at G as in any case.

Use this method on long stretches of work where most of the cross sections are level. Use a hand level to take those that are not level. For a level rod use a piece of measuring tape tacked to a strip of light wood ⅞" × 2". Support the hand level on the back of a knife blade stuck into, or cut into, a stick firmly set in the ground, or rest it on top of a square ended stick resting on a peg in the ground. Levels can then be taken with it and the rod, and cross sections staked out.

To set grade stakes on any but the center line, use the hand level to get the difference in elevation from the level peg at C_1 to a level peg at the place square out from C_1 on the line on which the grade point is to be found. Add this difference in elevation, with is sign, (+ up, — down) to C_1 D_1 to get the cut abreast of C_1 on that line. Verify the result by a second trial. In the same way get the fill abreast of C_2 on the same line. Use sticks the same as on the center line.

39. Where the center line crosses a slope at a small angle, as the fill approaches the hill side it may sometimes be possible to stake out a full fill cross section and yet there be a small cut outside of the upper side stake of the fill, Fig. 15. To find if this is so, measure out W for a cut on the up hill side of C. Take a rod reading there. If this rod reading is less than $+ K_C - H_C$ showing a cut there, complete the cross section, locate the grade point G, and record all the data, including surface lines.

40. **Earthwork Extension.** Sometimes it is necessary to extend the formation for the purpose of widening the roadbed. Figures 16 and 17 represent, in a general way, prismoids of the added formation between two successive cross sections. Fig. 16 is the case of a cut. Fig. 17 is the case of a fill. The extension of the road bed, or base, may be the same on suc-

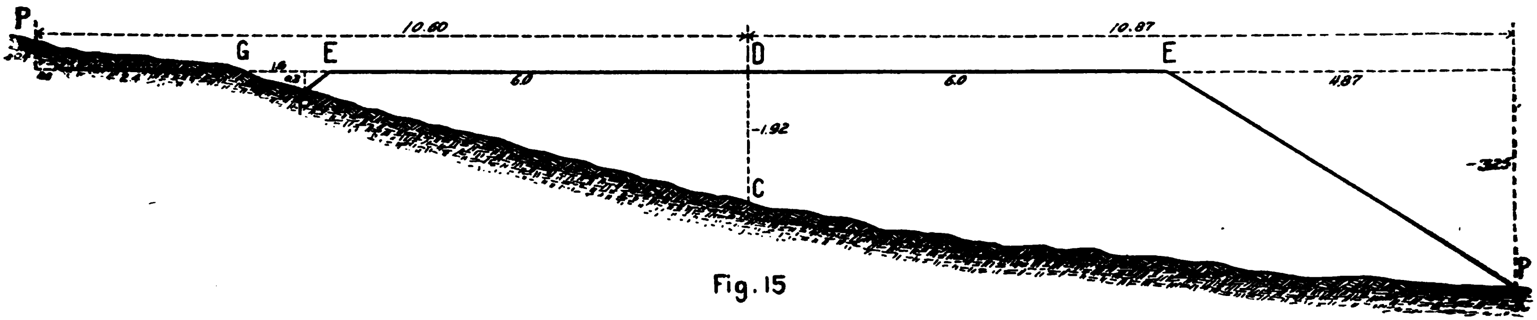


Fig. 15

cessive cross sections, as in Fig. 16, where $D'_0 E'_0 = D'_1 E'_1$, or they may be different, as in Fig. 17, where $D'_0 E'_0$ is greater than $D'_1 E'_1$. The slopes of the old work, D′ I, may be the same, or not. In any case connect the cross sections of the new work, with the old center line by measuring $D_0 D'_0$ and $D_1 D'_1$.

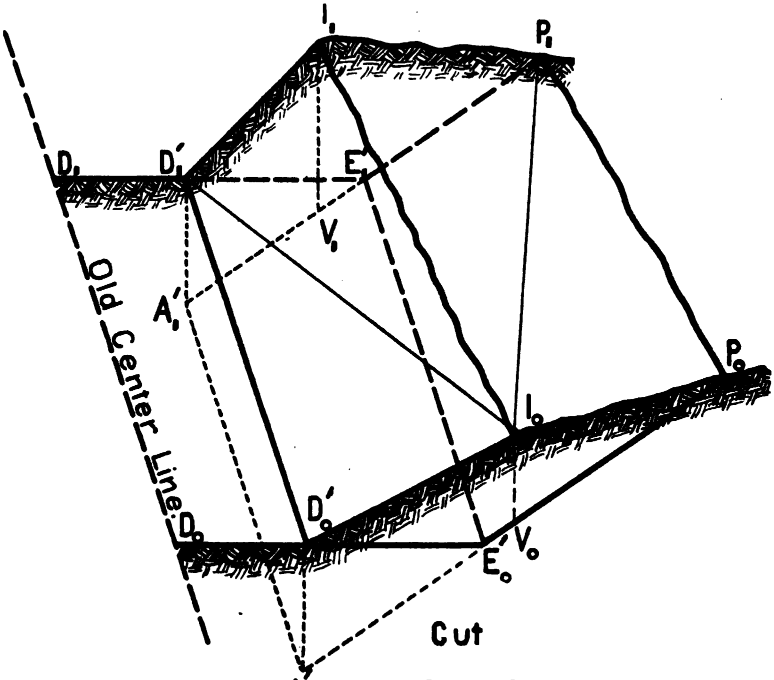


Fig. 16.

Cross section the work from the old center line. Treat D′ and I (the edges of the old slopes) as intermediates. Take slope readings, T, at these places with respect to the new slope E′ P. These will equal D′ A′ and I V. The computations of volume need not include anything between D and D′ but can begin at D′ and extend out to P. Record all surface lines, as I P on the surface of the ground, and D′ I on the old slope. Do not omit the surface line, D′ I, on the old slopes. If necessary, determine intermediates on the old slopes

between D′ and I and record all surface lines to them. There may also be intermediates between I and P. Be particular to get all necessary information, measurements, and data, including all surface lines. The surface lines should be taken, the same as in any other case. They will divide the ground, and the page of the Cross Section Book into triangles as usual, including the surface of the slopes of the old work.

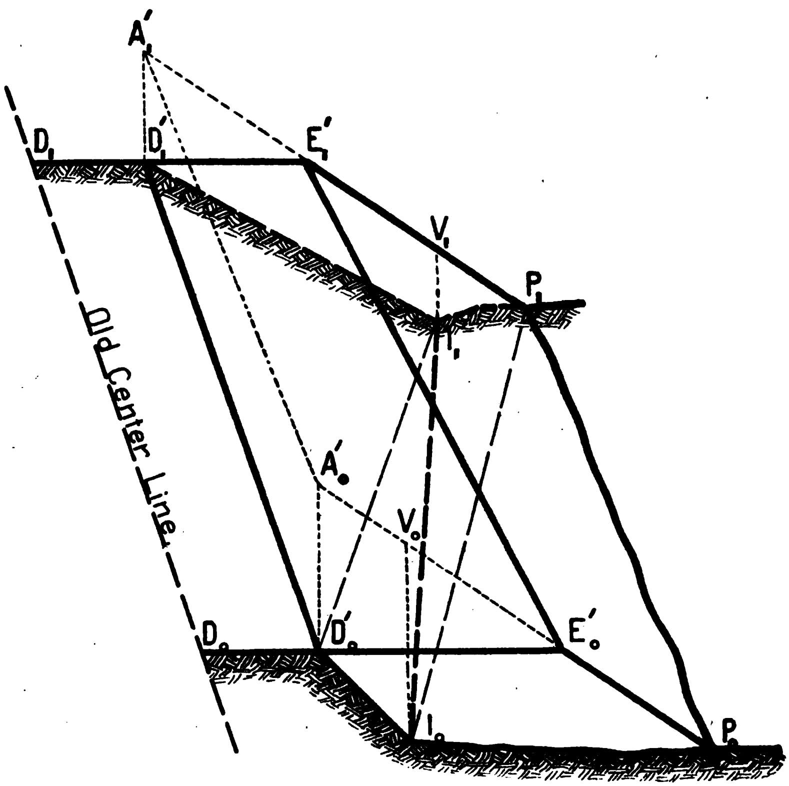


Fig. 17.

41. **Borrow Pits—Surveys.** Earth or rock may have to be removed for a variety of purposes. Such are to get filling material, or to form an excavation as in digging a cellar.

Such materials may be measured by measuring the pit from which they were taken. Frequently such pits are called Borrow Pits. There must be a survey of the surface of the ground where the pit is to be before any excavation begins, and of the bottom, slopes, or sides, and the top outline of the finished pit. The volume removed can be computed from the data gathered by these surveys.

Cover the ground where the pit is to be by a system of parallel lines. The spacing of these parallels should be some multiple of 27 feet, preferably 162 feet where the ground admits of it. The parallels should be near enough together so that surface lines between them will define the surface of the ground sufficiently well, the same as between cross sections of formations, as these parallels are the bases for cross sections of the pit. Connect the parallels with one or more base lines entirely outside of and beyond the pit, where there can be no possibility of their locations becoming lost, or destroyed. Read and record the angle the parallels make with the base line and measure and record their spacing along the base line. Connect this spacing measurement with some fixed mark on the base line that cannot be disturbed or lost. Mark the base line so there will be no possibility of losing its location or the positions of any marks on it. Safeguard this linear surface survey completely so it can be restored perfectly at any time.

Designate the parallels by letters of the alphabet, A, B, C, etc.

On the parallels set stakes at the breaks in the surface of the ground so near each other that straight lines between them will fit the ground well enough. Number these stakes on each parallel beginning with zero at the base line. Beginning at the base line measure and record the distances between these numbered stakes on each parallel. Keep a separate record for each parallel, as

Line F.	
0 to 1	28.3
1 to 2	51.6
2 to 3	79.8
&c.	&c.

Verify all these measursements, repeating them if necessary.

Extend this linear survey well beyond any possible limits for the pit. Pit surveys are frequently too small.

Map this linear survey on a large scale. Enter on the map all the numbers and measurements.

Take levels on the ground at each numbered stake. Record the levels for each parallel so they will appear separated in the record. Repeat these levels in full with a record in a separate place. Compare the elevations of the ground at the numbered stakes in the two records. If they do not agree well enough, repeat and verify the work till they do. Refer the levels to two or more bench marks entirely beyond any possibility of disturbance.

Enter the elevations on the large scale map at each numbered stake.

Take the map on to the ground and draw on it all the surface lines between the parallels, following the same rules as in recording surface lines between cross sections.

Ink the map in black.

The excavation should be finished by dressing it to true lines and plane surfaces.

Lay out the system of parallels again, covering the finished pit. If the bottom is a plane surface set the numbered stakes at their original spacing. Also set stakes on each parallel where it crosses the top edge and the bottom line of the slopès of the pit. Measure and record the positions of these stakes. Verify all of these measurements. Plat the positions of these stakes on the map. Take the map again upon the ground and draw on it the top and bottom lines of the slopes of the pit and any surface lines needed to define the surfaces of the finished pit. Be particular to get complete information regarding the locations of the outlines of the top and the bottom of every part of the slopes of the pit. Enter on the map all this added data. Ink the added information in another color.

Take levels at every stake on the restored parallels. Repeat these levels, and verify them as before.

Enter these latter elevations on the map in the new color. The map should now contain all the information needed to compute the volume of materials moved.

If the pit is not dressed to true lines it may still be measured. The same, or a different set of parallels referred to the same base line may be laid out over the pit. The two systems of parallels must be well connected. Stakes may be set at all breaks in the surface of the abandoned pit, and a surface survey made, recorded, mapped, and the surface lines drawn the same as for the surface of the ground. The levels should be run and elevations of the ground at all stakes obtained. Particular attention should be paid to getting complete information as to locations of outlines of pit, and its slopes. From the data gathered the volume of materials moved may be found by assuming a level plane within the pit and computing the parts above and below it., When this is contemplated, mark this plane on the sides of the pit where each parallel meets it. Include these stakes in the pit survey. If two sets of parallels are used the points where both sets meet this plane should be marked. Those for the upper set of parallels should be added to the survey of them, and those for the lower set to the survey of them.

42. Volumes—General Facts. The amount of earthwork is the product of three factors, thus giving a volume. These factors are, the length of the volume, the vertical breadths of the cross sections, and the level distances on the cross sections between the places at which the vertical breadths are taken. They should all be reckoned in the same unit of linear measure.

The length of the volume is the distance between the vertical, parallel, planes of the two cross sections which form the ends of the volume.

The vertical breadths of a cross section are the vertical distances from the surface of the ground to the opposite line which bounds the cross section. This opposite line is the broken line of intersection of the plane of the cross section with the bottom and sides of an excavation, or the top and

sides of a fill. These vertical breadths must be taken at each break in the ground surface line and at each break in the surface line of the finished work, on every cross section.

The number of these breadths may be much reduced, and also the work of computation, where the cross sections have the same level base, and equal side slopes. Such are the cross sections of ditches, highways, railroads, canal prisms, and similar formations. The slope lines of such cross sections are conceived of as being produced to meet in a vertical line through the center stake, above in a fill, below in a cut. These produced slope lines are regarded as the opposite lines bounding the cross sections. The vertical breadths are reckoned to them, at the center and side stakes, and at the breaks in the surface of the ground between these stakes, where rod readings were taken when staking out the cross sections. The cross section notes may be kept so as to give these vertical breadths by subtracting one entry from another, but in the older forms of keeping cross section notes, in which the elevation of the ground with respect to the level base, or road bed, is recorded, all these breadths must be computed. At the center stake, the distance to be added to the center height from the road bed, or base, is constant for fills, or cuts, and should be prepared at the first. It is equal to half the width of the road bed divided by the horizontal distance, in feet, corresponding to one foot rise of the side slope line. The breadth at the side stakes is zero. At any of the places between the center and side stakes, where elevations with respect to the road bed are recorded, the vertical breadths to the slope line may be computed, as follows: Add to the elevation from the road bed the same constant used at the center stake. From this sum, subtract the quotient found by dividing the distance of the place from the center stake by the level distance, in feet, corresponding to one foot rise in the side slope.

The distances on the cross sections between the places where the vertical breadths are taken show the spacing of these breadths on the cross sections, and are measured, and recorded, in the field, between the places noted in the survey of the cross sections.

With the older forms of cross section notes, the distances

of these places from the center stake are measured, and recorded, making it necessary to subtract the lesser from the larger of the distances of two of these places from the center stake in order to get the distance between them which is needed for use in these computations.

The cross section notes must show the surface lines between the two cross sections which form the parallel ends of the volume. This applies not only to the ground surface before the work is begun, but also to the surface of the completed work, unless it is finished to lines previously designated. These surface lines connect the places noted on the survey of one of these end cross sections with the places noted on the survey of the other. They must be shown in the record in that position where they will fit best the surface of the ground, or completed work, between the two cross sections, and define a volume bounded by straight lines, only. There must be at least one surface line running to each of the places noted in the survey of both cross sections. Those on the center and side lines, on the ground surface, are omitted from the notes because they are always in the same places.

43. **General Rule for Volumes.** Prepare the vertical breadths on both end cross sections, at the center stakes (if any) and at all other places noted in the survey of these cross sections. Take them in order across each cross section. The vertical breadths at the edges of work with side slopes are zero. The notes for these places must be used in making up the multipliers for the breadths.

For each vertical breadth take, as a multiplier of that breadth, the sum of the distances to the places where the next breadths each way from it were taken on the same cross section; and the distance, or distances (if any) between the places on the other cross section, where breadths are taken, which are joined by the surface lines to the place where the vertical breadth is taken for which the multiplier is being formed. Form multipliers, in this way, for each of the vertical breadths on both end cross sections.

Multiply each vertical breadth by its multiplier.

Sum these products. Divide this sum by 6.

Multiply the quotient by the length of the volume.

The result will be in terms of the cube of the linear unit used in the dimensions.

44. Formations. If the volume being computed is between cross sections with the same base and like slopes; subtract from the sum of the products after dividing by 6, the half width of the base, squared, divided by the level distance in feet corresponding to one foot rise of the slopes.

If the dimensions are in feet, and the volume is to be in cubic yards; divide each vertical breadth, and its multiplier, by 9, and divide the sum of the products then formed by 2, instead of 6. Multiply the quotient by the length in feet. The result will be in cubic yards.

If the volume being computed is between cross sections with the same base and like slopes, the dimensions in feet, and the volume to be cubic yards; divide each vertical breadth, and its multiplier, by 9; divide the sum of the products then formed by 2, instead of 6; and subtract from the sum of the products after dividing by 2, one twenty-seventh of the half width of the base, squared, and divided by the level distance in feet corresponding to one foot rise of the slopes. Multiply the remainder by the length in feet. The result will be in cubic yards.

The volumes of extensions of the formation are computed the same as the volumes of the regular formation on one side of the center line. Refer to Figs. 16 and 17. D′ A′ will be the first breadth. Form the products by the general rule between D′ and P. Sum these. Divide by 6. Subtract from this quotient the volume of the prismoid D'_0 A'_0 E'_0 E'_1 A'_1 D'_1. In Fig. 16 this will be a triangular prism. In Fig. 17 it will be a prismoid and its volume must be computed by Eq 15. The result will be the volume in terms of the cube of the linear unit used in the dimensions. If the dimensions are in

feet and the volume is required to be in cubic yards follow the process of the paragraph above, and, after dividing by 2, subtract one twenty-seventh of the olume of D'_0 A'_0 E'_0 E'_1 A'_1 D'_1 in cubic feet. The result will be in cubic yards.

Preliminary Estimates for comparing the amounts of earthwork on different lines of communication may be made by the help of Table XXX in Searles' Field Engineering as explained in Chapter XII p. 266. This will give rough approximations.

Another convenient method for making preliminary estimates by the aid of a moment planimeter is explained in a paper entitled The Moment Planimeter or Integrator by Professor Fred Morley to be found in the Proceedings of the Purdue Society of Civil Engineers for 1898. This is a very simple method for taking off approximate quantities by running the tracing point of the moment planimeter along the outlines of a cut, or a fill, as shown on the profile.

In making Preliminary Estimates for comparing the amounts of earthwork on different lines of communication certain calculations may be made that will give some idea of these amounts, based on certain general assumptions. The prismoids are taken to be all of the same length. The cross sections are assumed to be all level. The volumes near the grade points are omitted, only those between regular stations being considered. This makes the common length the regular station distance. Each cut and each fill is figured separately. In the case of a cut, or a fill (omitting the small ends) find twice the sum of the center cuts, or fills, and subtract from this sum the two end ones, forming term one. Find twice the sum of the squares of the center cuts, or fills, and subtract from this sum the squares of the two end ones, forming term two. Find the sum of the products of the center cuts, or fills, multiplying each one by the next, forming term three. Add these three terms. Do the same for all the cuts and all the fills, if both are to be computed. Add all these results. The total sum will show the relative amount of earthwork on a line, or a piece of a line, when compared with

a like sum from another line. These sums are not volumes. If two lines are nearly alike, it will be necessary to multiply the first term by six times the width of the road bed and the sum of the last two terms by twice the number of feet level distance corresponding to one foot rise of the slopes. Then find the total sums of these products for each line, and a closer comparison can be made. These sums are not volumes. These comparisons are only rough approximations, being based on certain general assumptions that will not be true in fact, and may not give close comparisons. These can only be obtained from the cross section notes by computing the actual volumes.

45. **Borrow Pits—Volumes.** Borrow pits may be excavated with uneven slopes. The surface lines on these slopes must be recorded between the places noted on the successive parallels, in the survey of the bottom of the pit, the same as elsewhere on the bottom of the pit. That is, the survey, and record, of the bottom of the pit must extend to the top of the slopes, at the surface of the ground. The slope on any parallel section runs from the top of the ground to the next point of survey on that section. If the bottom of the pit is irregular, and the surface lines, and points of survey, are not in the same vertical planes, or lines, as those on the top of the ground, the volume must be divided into two parts by a level plane, and these parts computed separately. It is best to assume the elevation of this plane when surveying the bottom of the pit, and mark its intersections with each of the parallel sections of the pit survey for both the top of the ground and the bottom of the pit. Then use these marks as points of survey on the parallel sections, and record surface lines to them. By doing this, there will be but one surface line on the slope of the prismoid between a pair of parallel sections. This fact reduces all the conditions to two cases. First, when there is a surface line on the top of the ground in the same vertical plane with the slope surface line; second, when there is not.

Before applying the rule, find the vertical breadth at the bottom of the slope line on each parallel section, and its distances from the next breadths each way from it; also draw the surface line to it. Include this data in the computations.

First—When there is a ground surface line in the same vertical plane with the slope surface line. This is always the case when but one plane of the ground surface is cut by the vertical plane of the slope surface line. The intersection of these planes is to be taken as a ground surface line; a breadth figured for the end of it that is not on the slope; and the distances found from this breadth to those next each way from it. Then the volume is to be computed, out to the top edge of the pit, by the first section of the rule.

Second—When there is not a ground surface line in the same vertical plane with the slope surface line. Consider a plane passed through the line of the flatter slope and the point at the bottom of the steeper slope. Let the plane of this flatter slope extend to meet a vertical plane through the ground surface line at the top of the slope. Compute a breadth from the top of the steeper slope to this plane of the flatter slope. Include this breadth, and compute the volume, by the first section of the rule, except as follows: Form the multiplier for this breadth by the rule, but subtract from this multiplier the sum of the distances covered by the slope lines of both parallel sections, if the slope surface line runs to the top of the steeper slope, or, subtract the sum of the distances covered by the steeper slope, only, if the slope surface line runs to the top of the flatter slope. With this remainder as a multiplier, proceed by the rule. If this last named multiplier is minus, subtract the figures in the product formed by it from the sum of the other products, before dividing by 6 and multiplying by the length of the volume.

Parallel sections may be used over the greater part of a pit, but owing to its broken outline, there may remain considerable portions of it, outside of the parallels, or at their ends, to which the above described processes cannot be applied without much labor. The volume of these parts may be computed by division into truncated triangular prisms, having vertical edges at the vertices of the triangles into which the surface lines divide the bottom of the pit, or the surface of the ground; or both, where a level plane is employed to divide the whole volume into two parts. The vertices of these tri-

angles may be located by offsets from a straight line with their distances apart, on that line. The volume of a truncated triangular prism equals the product of one-third the sum of its edges by the area of its right section. To find the area of the right section, multiply the distance between the first and second offsets to the vertices of the triangle forming one end of the prism, by half the difference between the second and third offsets; and multiply the distance between the second and third offsets by half the difference between the first and second offsets. Add the two products for the area of the right section.

In some cases a rectangle, traversed by parallel sections may be surveyed from the central portion of a pit and the borders computed by division into prisms.

Some pits are so irregular that the prism method should be applied over the whole of them. The required points of survey may be determined by reference to a system of parallel lines, with offsets.

See Henck's Field Book for Engineers, sections 114 to 117, also Johnson's Surveying section 311 for detailed accounts of the prism method. See Johnson's Surveying section 312 and Raymond's Surveying, sections 266 to 268, for some suggestive methods of estimating volumes approximately (but usefully) from contour maps.

46. Corrections on Curves. At each regular station on curves a volume is included twice on the inside of the curve and another volume omitted entirely, on the outside of the curve. These volumes depend upon the degree of the curve and the form and dimensions of the cross section at any station. The volume on the inside must be subtracted from the computed volume, and the volume on the outside added. The difference of these two volumes is the correction required, and it may be plus if the outside volume is the greater, or minus if the inside volume is the greater. At the P. C. and P. T. use half this correction. At a P. C. C. use the half sum, and at P. R. C. the half difference of the corrections for the curves

which meet at these places. These corrections are not needed, unless they will materially affect the price for the work.

Use the same breadths and their distances apart on the cross section, as when computing the volume of the prismoids of the formation. Take them in order from the center stake out to the side stake.

Form two multipliers for each breadth. The first is ½ its distance from the center stake plus ⅙ the distance to the next breadth outside of it. The second is ½ its distance from the cented stake minus ⅙ the distance to the next breadth inside of it. There is no second multiplier for the breadth at the center stake. Form no multiplier for the breadth at the side stake, as this breadth is zero.

Multiply the breadth at the center stake by its multiplier, and the other breadths by each of theirs.

Number these products, each for the breadth to which it belongs. Record these products in separate columns marked 1st and 2d, according to which multiplier gave them.

From these products, form a multiplier for each distance between breadths, proceeding from the center stake outwards, by adding the 1st product for the first breadth to the 2d product for the second breadth, at the ends of that distance.

Multiply each distance by its multiplier.

Sum these products.

Multiply the sum by the sine of the degree of the curve.

The result will be the volume on one side of the center stake.

Make the same computations for the other side of the center stake.

Call the volume on the outside of the curve plus, and that on the inside minus.

Take their sum, with its sign, for the required correction.

In side hill work compute the correction for the cut separate from the correction for the fill.

In side hill work compute the correction for the side which is the same as the center stake, either cut or fill, as above directed. For the other side include the space between the roadbed and slopes the same as before. Consider the side

stake to be at the edge of the roadbed. Compute the vertical distance from the roadbed to the slope produced, at the place of the grade point on this cross section, and use this vertical distance as a breadth in the computations. Make the computations as above directed. Give the volumes their signs for the outside and inside of the curve. Take their sum, with its sign for the required correction.

In side hill work treat the side which is different from the center stake, either cut or fill, as above first directed, except that the breadth at the center stake will be the distance from the roadbed to the intersection of the slopes, and the breadth at the grade point will be the vertical distance of the roadbed, at that point, from the slope. Subtract from the volume found, $\frac{1}{6}$ of the cube of half the width of the roadbed multiplied by the sine of the degree of the curve and divided by the horizontal distance in feet corresponding to one foot rise of the slope. This will give the volume from the roadbed and not from the intersection of the slopes. Make this calculation for but one side. The result will be the correction for the cut or the fill, according to which is computed, plus for the outside of the curve, minus for the inside.

For three level cross sections the correction equals one-sixth the breadth at the center stake to the intersection of the slopes, multiplied by the sum of the distances from the center to the side stakes; and this by the difference of these distances; and this by the sine of the degree of the curve. If the distance out on the inside of the curve is the larger, the result is minus, if the smaller, the result is plus.

47. Work Done. Work which has been done should be cross sectioned and the surface lines noted the same as in the survey of the bottom, sides, and top outline, of a completed borrow pit. The surface lines where the work done joins the natural surface of the ground must be noted. The volumes may then be computed by the rules for borrow pits.

If there is a record of a survey of the original surface of the ground and the two surveys can be connected the correct volume can be found. If the work done is on a line of communication, as a highway, canal, or railway, and the new survey can be connected with the original survey, the center

line should be rerun in its former position, and the new center stakes set in the same vertical with the former ones.

A level plane, as suggested for borrow pits in certain cases, can be often used to advantage.

Breadths should be found at the breaks in the surface of the work, between that surface and the surface of the ground, or to the level plane. On a transportation line these breaks will be at the edges of the roadbed, or base, and possibly elsewhere. After finding these breadths with surface lines to them, in the way directed for borrow pits, include them in the computations. On a transportation line do not attempt to figure to the slope lines produced, unless the slopes are found to be alike, the work dressed to true lines, and the center line to be in the center of the roadbed.

There are two other cases. In these only approximate estimates can be made.

The first of these is when the ground surface lines are lacking; the record of the original survey, including levels, is preserved; and the two surveys can be connected. Levels may have been taken so frequently that the resulting volume will be practically correct. Get the vertical breadths,—or heights,—at the breaks in the surface of the work, as accurately as possible, to the surface of the ground or to an assumed level plane. If a level plane is used, the breadths between it and the surface of the ground must be found at all the places where levels were taken. Find the positions and spacings, horizontally, of all the breadths, so the area of a right section of a prism which has three, or four,—as the case may be—of these breadths for its edges, can be computed. Map all this data so the items will appear plainly in their proper relative positions with respect to each other. The map will furnish a good place to record results of computations. It will also enable the work of computation to be kept track of much more readily. Having prepared such a map and entered all the data on the same—verifying every entry—employ the method of truncated triangular or quadrangular prisms for computing the volumes. That is, multiply the right section of each prism by the arithmetical mean of the heights at its edges. Enter the results on the map in the space for each prism. Sum the results for the total volume.

This method by prisms, or some modification of it, must

be used when the surface lines on the ground are unknown, as there is no way of telling how they were situated, and no possible combination of the items on the map is likely to be as trustworthy, or as easy to compute, as that obtained by prisms.

The second case is when the ground surface lines and former levels are wanting; or the old survey and the new one cannot be connected. The new survey will show the surface of the work done and the limiting outlines of it, with levels at all breaks in the surface and in the limiting outlines. The surface of the ground must be taken as extending across the work joining the breaks in the outline of the work where it joins the natural surface. If the work is of some extent no sort of a guess can be made as to the shape in relief of the surface of the ground which has been destroyed, or buried, by the work. The best that can be done is to consider the work in sections, using a level plane through the lowest point of the ground surface outline in figuring excavation, or through the highest point of the ground surface outline in figuring filling. The part between this level plane and the surface of the work can be correctly estimated by prisms. The part between this level plane and the original surface of the ground should be computed as nearly as practicable by prisms, studying the data carefully. The division of the whole work into sections for purposes of calculation should be made with consideration and judgment. A map of the data is particularly useful in this case.

48. Formulas of General Application.

A_0 = area of one of the parallel plane ends of a prismoid.

A_1 = area of the other end.

M_1 = area of a plane section of the prismoid, midway between, and parallel to A_0 and A_1.

L_1 = the distance apart of A_0 and A_1, which is the length of the prismoid.

In earthwork, A_0 and A_1 are the cross sections surveyed when staking out a highway, a canal, a ditch, a levee, an embankment, or any form of earthwork where the length of the work greatly exceeds its breadth. The notes for these

cross sections are to be found in the Cross Section Book. L_1 is the distance along the center line of the work, from C_0, the center stake on A_0, to C_1, the center stake on A_1, shown by the station and plus numbers in the Cross Section Notes.

In the case of borrow pits, cellars, reservoirs, grading of land, filling in, dredging, or where the nature and outline of the work is such that a system of parallels is used in making the surveys, A_0 and A_1 are the sections of the work on two successive parallels, and L_1 is the distance between them. The notes for these parallels are gathered from the various surveys and brought together on a map, as described for use in surveying borrow pits.

V_1 = the volume of the prismoid between A_0 and A_1.
b_0 = the vertical breadth of A_0 at any break in its outline, found from the cross section notes.
b_1 = any vertical breadth on A_1.
d_0 = the level distance between any two breadths on A_0.
d_1 = the level distance between any two breadths on A_1.
$\Sigma\, d_0$ with respect to b_0 = the sum of the two distances on A_0 that meet at b_0.
$\Sigma\, d_1$ with respect to b_1 = the sum of the two distances on A_1 that meet at b_1.
$\Sigma\, d_0$ with respect to b_1 = the sum of all the level distances between breaks in the outline of A_0 which are connected by surface lines to the break in A_1 where b_1 is taken.
$\Sigma\, d_1$ with respect to b_0 = the sum of all the level distances between breaks in the outline of A_1 which are connected by surface lines to the break in A_0 where b_0 is taken.

The surface lines, distances and connections are shown in the records of the surveys.

$\Sigma\, (b\, \Sigma\, d)$ on any section = the sum of these products for all the breadths of that section.
$\Sigma\, b_0\, (\Sigma\, d_0 + \Sigma\, d_1) + \Sigma\, b_1\, (\Sigma\, d_1 + \Sigma\, d_0)$ for any prismoid = the sum of these products for all the breadths on both A_0 and A_1.

Take the breadths in succession, and in order, across a section when computing.

The above notation may be made to apply to any prismoid, whatever its number, by changing the subscripts to correspond to that number.

A_0, A_1, A_2, A_3, and so on to A_n, represent the areas of the successive sections of the work.

L_1 being the distance apart of A_0 and A_1; L_2 would be the distance between A_1 and A_2; L_3 between A_2 and A_3; and so on to L_n, between A_{n-1} and A_n.

Similarly for other symbols.

(S), with suitable subscripts within and without the ( ) = area of plane sections of a prismoid, parallel to its ends, in special cases, as at one-fourth, or one-eighth, of its length from A of the smaller number. $(S_3)_4$ would be in the volume V_3, between A_2 and A_3 at $\frac{1}{4}$ L_3 from A_2.

(V), with suitable subscripts within and without the ( ) = the volume of that part of a prismoid between A of the smaller number and (S) having the same subscripts as (S).

$(V_3)_4$ would be the volume between A_2 and $(S_3)_4$ in the prismoid between A_2 and A_3, the length of which from A_2 to $(S_3)_4$ is $\frac{1}{4}$ L_3.

X, with suitable subscripts = area of any plane section of a prismoid, taken parallel to its ends.

x, with suitable subscripts = distance of X from A of the smaller number.

$(V)_x$ = volume between A of the smaller number and X. Subscripts may be used within the ( ).

49. Areas of Sections.

1. $A_0 = \Sigma\,(b_0\ \frac{1}{2}\ \Sigma\, d_0)$
2. $A_1 = \Sigma\,(b_1\ \frac{1}{2}\ \Sigma\, d_1)$

For other sections, first compute,

3. $K_0 = \Sigma\,(b_0\ \Sigma\, d_1)$ and
4. $K_1 = \Sigma\,(b_1\ \Sigma\, d_0)$

In a prismoid of a length L_1 and Volume V_1, but having a uniform right section, the area of this right section would be

$$5.\quad \frac{V_1}{L_1} = \tfrac{1}{6}\,(2A_0 + 2A_1 + K_0 + K_1)$$

$$6.\quad = \tfrac{1}{6}\left[\Sigma\left[b_0\,(\Sigma\, d_0 + \Sigma\, d_1)\right] + \Sigma\left[\,b_1\,(\Sigma\, d_1 \times \Sigma\, d_0)\,\right]\right]$$

$$7.\quad M_1 = \tfrac{1}{4}\,(A_0 + A_1 + K_0 + K_1)$$

$$8.\quad = \tfrac{1}{4}\left(6\,\frac{V_1}{L_1} - A_0 - A_1\right)$$

$$9.\quad X. = \frac{x_1^2}{L_1^2}\left(A_0 + A_1 - K_0 - K_1\right) - \frac{x_1}{L_1}\left(2A_0 - K_0 - K_1\right) + A_0$$

$$10.\quad = A_0 - \frac{x_1}{L_1}\left(A_0 - A_1\right) - \frac{x_1}{L_1}\left(1 - \frac{x_1}{L_1}\right)\left(A_0 + A_1 - K_0 - K_1\right)$$

$$11.\quad = A_0 - \frac{x_1}{L_1}\left(A_0 - A_1\right) - 2\,\frac{x_1}{L_1}\left(1 - \frac{x_1}{L_1}\right)\left(A_0 + M_1 + A_1 - M_1\right)$$

$$12.\quad = A_0 - \frac{x_1}{L_1}\left(A_0 - A_1\right) - 3\,\frac{x_1}{L_1}\left(1 - \frac{x_1}{L_1}\right)\left(A_0 - \frac{V_1}{L_1} + A_1 - \frac{V_1}{L_1}\right)$$

$$13.\quad = A_0 - \frac{x_1}{L_1}\left(A_0 - A_1\right) - 12\,\frac{x_1}{L_1}\left(1 - \frac{x_1}{L_1}\right)\left(\frac{V_1}{L_1} - M_1\right)$$

x_1 may be taken as any fractional part of L_1

For $x_1 = \frac{3}{7} L_1$

$$14.\quad X_1 = A_0 - \tfrac{3}{7}\,(A_0 - A_1) - \tfrac{12}{49}\,(A_0 + A_1 - K_0 - K_1)$$

As the coefficients of but two terms in the value of X_1 are changed, it is not a difficult matter to obtain successive values of X_1 whenever needed.

50. Volumes.

$$15.\quad V_1 = \tfrac{1}{6} L_1\,(A_0 + A_1 + 4M_1)$$

$$16.\quad = \tfrac{1}{6} L_1\,(2A_0 + 2A_1 + K_0 + K_1)$$

$$17.\quad = \tfrac{1}{6} L_1\left[\Sigma\left[b_0\,(\Sigma\, d_0 + \Sigma\, d_1)\right] + \Sigma\left[b_1\,(\Sigma\, d_1 + d_0)\right]\right]$$

$$18.\quad (V_1)_{x_1} = x_1\left[\frac{x_1^2}{L_1^2}\,\tfrac{1}{3}(A_0 + A_1 - K_0 - K_1) - \frac{x_1}{L_1}\,\tfrac{1}{2}\,(2A_1 - K_0 - K_1) + A_0\right]$$

$$19.\quad = x_1\left[A_0 - \tfrac{1}{2}\frac{x_1}{L_1}(A_0 - A_1) - \frac{x_1}{L_1}\left(1 - \tfrac{2}{3}\frac{x_1}{L_1}\right)\tfrac{1}{2}(A_0 + A_1 - K_0 - K_1)\right]$$

$$20.\quad = x_1\left[A_0 - \tfrac{1}{2}\frac{x_1}{L_1}(A_0 - A_1) - \frac{x_1}{L_1}\left(1 - \tfrac{2}{3}\frac{x_1}{L_1}\right)(A_0 - M_1 + A_1 - M_1\right]$$

$$21.\quad = x_1\left[A_0 - \tfrac{1}{2}\frac{x_1}{L_1}(A_0 - A_1) - \frac{x_1}{L_1}\left(3 - 2\frac{x_1}{L_1}\right)\tfrac{1}{2}\left(A_0 - \frac{V_1}{L_1} + A_1 - \frac{V_1}{L_1}\right)\right]$$

$$22.\quad = x_1\left[A_0 - \tfrac{1}{2}\frac{x_1}{L_1}(A_0 - A_1) - \frac{x_1}{L_1}\left(3 - 2\frac{x_1}{L_1}\right)2\left(\frac{V_1}{L_1} - M_1\right)\right]$$

x_1 may be taken as any fractional part of L_1

For $x = \frac{7}{11} L_1$.

$$23.\quad (V_1)x_1 = \tfrac{7}{11} L_1\left[A_0 - \tfrac{7}{22}(A_0 - A_1) - \tfrac{133}{726}(A_0 + A_1 - K_0 - K_1)\right]$$

As the coefficients of but two terms in the paranthesis and the fraction in the coefficient of the parenthesis are changed, it is not a difficult matter to obtain successive values of $(V_1)x_1$, whenever needed.

51. Center of Gravity Equations. In earthwork usually but one of the coordinates of the center of gravity is needed. This is the one in the direction of the center line of the work, or in a line perpendicular to the plane sections of the work.

For a given prismoid, between A_0 and A_1

From the end A_0.

$$24.\quad x_g = \frac{1}{6V_1} L_1^2 (A_1 + 2M_1)$$

$$25.\quad = L_1 \frac{A_1 + 2M_1}{A_0 + A_1 + 4M_1}$$

$$26.\quad = \tfrac{1}{2}L_1 - \tfrac{1}{2}L_1 \frac{A_0 - A_1}{A_0 + A_1 + 4M_1}$$

$$27.\quad = \tfrac{1}{2}L_1 - \tfrac{1}{2}L_1^2 \frac{A_0 - A_1}{6V_1}$$

$$28.\quad = \tfrac{1}{2}L_1 - \tfrac{1}{2}L_1 \frac{A_0 - A_1}{6\dfrac{V_1}{L_1}}$$

The distance of the center of gravity from the mid section M_1, is

29. $$-\tfrac{1}{2}L_1\frac{A^0 - A_1}{A_0 + A_1 + 4M_1} = -\tfrac{1}{2}L_1\frac{A_0 - A_1}{6\frac{V_1}{L_1}}$$

Of course this distance will be towards the larger end of the prismoid from the mid section, M_1.

From the end A_1.

30. $$L_1 - x_g = \frac{1}{6V_1}L^2_1(A_0 + 2M_1)$$

31. $$= L_1\frac{A_0 + 2M_1}{A_0 + A_1 + 4M_1}$$

32. $$= \tfrac{1}{2}L_1 - \tfrac{1}{2}L_1\frac{A_1 - A_0}{A_0 + A_1 + 4M_1}$$

33. $$= \tfrac{1}{2}L_1 - \tfrac{1}{2}L_1^2\frac{A_1 - A_0}{6V_1}$$

34. $$= \tfrac{1}{2}L_1 - \tfrac{1}{2}L_1\frac{A_1 - A_0}{6\frac{V_1}{L_1}}$$

If one of these forms be divided by the other, the ratio of the distances of the center of gravity from the two ends of the prismoid is obtained.

35. $$\frac{x_g}{L_1 - x_g} = \frac{A_1 + 2M_1}{A_0 + 2M_1}$$

36. $$= \frac{A_1 - A_0 + 6\frac{V_1}{L_1}}{A_0 - A_1 + 6\frac{V_1}{L_1}}$$

Part of a given prismoid between the end A_0 and any section X_1.

From the end A_0

37. $$x'_g = \tfrac{1}{2}x_1 - \tfrac{1}{2}x_1^2 \frac{A_0 - X_1}{6\,(V_1)_{x_1}}$$

Part of a given prismoid between any section X_1 and the end A_1.

From the end A_0.

38. $$x''_g = x_g + (x_g - x'_g)\,\frac{(V_1)_{x_1}}{V_1 - (V_1)_{x_1}}$$

From this expression it appears that the center of gravity of that part of a given prismoid between X_1 and A_1 is beyond the center of gravity of the whole prismoid, reckoned from A_0, a distance equal to

39. $$(x_g - x'_g)\frac{(V_1)_{x_1}}{V_1 - (V_1)_{x_1}'}$$

In the case of any two prismoids, the ratio of the distances from their mid sections to their centers of gravity can be written.

40. $$\frac{(x_g - \frac{1}{2}L)_1}{(x_g - \frac{1}{2}L)_n} = \frac{L_1^2}{L_n^2} \times \frac{V_n}{V_1} \times \frac{A_0 - A_1}{A_{n-1} - A_n}$$

This may be applied to the parts of a given prismoid. For the part between A_0 and X_1, whose length is x_1, the distance of its center of gravity from its mid section at $\frac{1}{2}x_1$ may be found from the following proportion:

41. $$\frac{x'_g - \frac{1}{2}x_1}{x_g - \frac{1}{2}L_1} = \frac{x_1^2}{L_1^2} \times \frac{V_1}{(V_1)_{x_1}} \times \frac{A_0 - X_1}{A_0 - A_1}$$

From these last two equations it appears that after computing the distance of the center of gravity of a prismoid from its mid section, the corresponding distance for any other prismoid whose volume is known, may be found without computing $A_0 + A_1 + 4M_1$ for that prismoid. This may be useful in some instances.

52. Work Equations. The work of moving a prismoid from its center of gravity to its mid section is found by multiplying the distance of the center of gravity from the mid section by the volume.

For a given prismoid the result is

42. $-\frac{1}{12} L^2_1 (A_0 - A_1)$

For a part of a given prismoid between A_0 and X_1 the result is

43. $-\frac{1}{12} x^2_1 (A_0 - X_1)$

Dividing the latter of these terms by the former a fraction is formed that may be used to multiply the work by, in the case of the whole prismoid, to get the work of moving the part to its mid section. This fraction is.

44. $\frac{x_1^2}{L_1^2} \frac{A_0 - X_1}{A_0 - A_1}$

From Eq. 9

$$A_0 - X_1 = \frac{x_1}{L_1}(2A_0 - K_0 - K_1) - \frac{x^2_1}{L^2_1}(A_0 + A_1 - K_0 - K_1)$$

Therefore X_1 need not be computed.

From the above expressions for work it appears that for a number of successive prismoids of equal length, L, their cross sections being A_0, A_1, A_2, etc., to A_n, the sum of the terms of work for moving them all from their respective centers of gravity to their mid sections will be

45. $-\frac{1}{12} L^2 (A_0 - A_n)$

In computations involving work these volumes may be treated as if situated at their mid sections instead of at their centers of gravity and the total result relating to them corrected by the above expression.

53. Suggestions. In computing earthwork if only the volume is required use Eq. 17. If other computations are to be made with respect to a prismoid it will be found more convenient to first form the four products represented by Eq. 1,

2, 3, and 4, and to employ these values in the subsequent work. as the equations involving these values may be seen to be usually simpler than others in the same group.

In computations relating to the moving of earth use expressions 42, 43, 44, and 45. This will save the work of computing the positions of the centers of gravity of the prismoids at all. They are then to be treated as if situated at their mid sections instead of at their centers of gravity. Observe the signs of the various items of work represented by 42, 43, 44, or 45.

It may be required to cut off a given volume from the end of a given prismoid.

If there is no difference in the areas of the end cross sections, that is $A_0 = A_1$, ordinarily the value of x_1, the distance from A_0 to where the required volume ends, may be found by simple proportion.

$$46. \quad x_1 = L_1 \frac{(V_1)_{x_1}}{V_1}$$

In the case of the double wedge, and similar prismoids, this would not be true, as A_0 may equal A_1, many times both being zero, while the mid section may be large. The prismoids in ordinary earthwork do not belong to this class.

If A_0 and A_1 differ somewhat, a serviceable estimate of x_1 may be made by using 46 and allowing something for $A_0 - A_1$. After a few trials, perhaps no more than two, an idea of the change in $(V_1)_{x_1}$ corresponding to any designated change in x_1, may be obtained that will enable the right value of x_1 to be selected. These trials are the more easily made because the indivdiual terms, within the parenthesis in the equations for $(V_1)_{x_1}$ are changed simply by changing the value of x_1 and the fraction $\frac{x_1}{L_1}$ as coefficients, without recomputing the whole of any term. As but two terms in the parenthesis are changed, the third being A_0, the changes can be rapidly made after a little experience. The method here outlined will be found much more practicable than any attempt to solve the four term cubic equation which expresses the value of $(V_1)_{x_1}$.

54. **Haul.** Contracts for the movement of earth, in constructing lines of travel, frequently name a price per cubic yard for all distances not exceeding a certain limit, say 300 or 400 feet. For greater distances an extra price is allowed, depending upon the excess of the distance over that for which the price does not vary. The terms Long Haul, Extra Haul, and Over Haul, are applied to this with no great precision, as sometimes the excess distance is referred to, sometimes the cubic yards of earth to which the extra price applies, and sometimes the amount of money involved, rarely, if ever, referring to the extra work the excess distance makes. As the extra work is different for every cubic yard moved it becomes necessary to find the average Haul for the entire volume to which the extra haul applies. In some of the estimates relating to the moving of earth the average haul for any designated volume, in any situation, may be needed. The average haul for a volume of earth is the distance between the centers of gravity of the volume moved, and the volume it makes.

With a designated free haul limit, say 300 or 400 feet, as above noted, and where the cuts must go into the fills, the limit of free haul, or short haul, as it is sometimes called, may be noted during the progress of the work, in some few cases. From this, with cross sections at its ends, with measurements of the earth afterwards moved, and of the fill it makes, the extra haul may be obtained. With proper surveys and notes this can be done better in the office. The calculations relating to average haul involve positions of centers of gravity. In earthwork it is only the center line position, or the distance from one end of the prismoid, that is needed. No attention is paid to the other two co-ordinates.

In order to complete a certain portion of a fill from the end of a cut, or to find out what part of a fill will be required to receive a certain portion of a cut, the volumes of the prismoids may be balanced off against each other till one will be able to tell between what two cross sections the work, in either case, must end. If these cross sections differ but slightly in area, the position of the limiting cross section may be quite nearly located by simple proportion. If this is not sufficiently exact or the areas of the cross sections differ considerably use the method of equations 18 to 23, following

the suggestions given after equation 46. Having found the position of the limiting cross section, include this last prismoid in the calculations for extra haul.

It may be required to divide a cut or a fill into two parts such that the work in each shall be the same. If a cut is all to go into the fills at its ends the question may arise as to where the hauling out at either end shall cease. To answer this the volumes of the prismoids from each grade point in the cut may be balanced off against the adjoining fills at the ends of the cut until all the earth in the cut is provided for, and the volumes going out at each end, multiplied by their average hauls are equal. Sometimes a fill is to be made from two, or more, cuts, or borrow pits, and it is desirable to know how much should be taken from each. This may be determined in a similar way. Such questions as these last are not likely to arise where the work is uniform and light. In a mountainous, rough, or rocky region, they may become important.

An accurate estimate of the average haul, or the extra haul, or of the work of moving a mass of materials, may be made by computations based on the field notes of the cross sections, and other surveys. As this is rarely necessary other methods are used.

To make the computations above referred to, first determine the positions of the centers of gravity of the various prismoids by equations 24 to 29. The center of gravity of any number of prismoids may then be determined as follows, graphically. The same thing can, of course, be done by computation, using the principle of moments; by assuming any point on the center line of the work as an origin, multiplying the volume of each prismoid by the distance of its center of gravity from the origin, and dividing the sum of these products by the sum of the volumes; the quotient showing the distance of the center of gravity of the entire mass from the origin. The graphical process may be carried out on cross ruled paper that is correctly spaced, such as profile and cross section paper. Lay off, or count off, on a straight line, representing the center line of the work, to a suitable scale, the lengths of the prismoids in their order of succession. Plat the positions of the centers of gravity of the prismoids at their

proper places in these spaces. At the center of gravity point of the first prismoid, count off an ordinate above the center line equal to the volume of the second prismoid, using any suitable scale. At the center of gravity point of the second prismoid, count off an ordinate below the center line equal to the volume of the first prismoid, using the same scale for volumes. Fit a straight edge to the marks at the ends of these ordinates. Mark the point where the straight edge cuts the center line. This will be the center line position of the center of gravity of these first two prismoids. Count off an ordinate above the center line from the center of gravity of the first two prismoids, equal to the volume of the third prismoid, using the scale of volumes. At the center of gravity of the third prismoid count off an ordinate below the center line equal to the volume of the first two prismoids. Connect the ends of these last two ordinates by a straight edge, and mark its intersection with the center line. Here count off an ordinate above the center line equal to the volume of the fourth prismoid. At the center of gravity of the fourth prismoid count of an ordinate below the center line equal to the sum of the volumes of the first three prismoids, and continue the work as before. Thus proceed till the volume of the last prismoid has been laid off above the center line, and connected by the straight edge with the corresponding ordinate below the center line and the intersection of the straight edge with the center line marked. This last mark shows the center line position of the center of gravity of all the prismoids embraced in the process. Where the straight edge cuts the center line is the center of gravity of all the prismoids used in reaching that point. Volumes may be added to or left off of this construction. This adapts it for use in connection with the solution of the problem of average haul.

A graphical method for solving the problems relating to haul on lines of communication, and similar works is outlined below. It gives results quite near enough to the truth for all ordinary cases. It is convenient to use, and can be made to answer a variety of inquiries. Only an elementary presentation of this method is made here. In Molitor and Beard's Manual for Resident Engineers, section sixty-five, may be found an application of the principles of diagrams of this kind to the details of construction.

Use profile paper, or that which is accurately cross ruled. Choose a horizontal line to represent the center line of the work. Assume a scale for distances along that center line. Mark on the center line the position of a center line grade point, G, where a cut and fill join. Mark on the center line the positions of all cross sections. On the cut side of G at the location of each cross section count off an ordinate equal to the total volume from G to that cross section, using some suitable scale of volumes. Mark the ends of all these ordinates. Join these marks in succession, beginning with G, by straight, fine, black, full, lines. This broken line is the Volume Curve of excavation from G. This curve is usually drawn below the center line.

Beginning at G, but extending on the opposite side from the cut, construct the volume curve for embankment in the same manner as the one for excavation was constructed. This volume curve should be on the same side of the center line as the excavation curve is. The volume curve for embankment should be inked in red, or a different color from that used on the cut side of G.

These two volume curves extending each way from G form a Haul Curve.

The distance between the two volume curves on any horizontal line is the limit of haul for the amount of materials indicated by the vertical distance from the center line at which the horizontal distance is taken. The horizontal distance is to the scale of the center line. The vertical distance is to the scale of volumes.

If any distance, as 300 feet, 500 feet, or 1,000 feet, be marked on the straight cut, or folded, edge of a piece of paper by placing it along the center line of the haul curve, this piece of paper may be used to determine the amount and position of the materials that need not be hauled a greater distance than that marked off. Keep one of the marks on the excavation curve and find where the other mark will fit the embankment curve when the edge of the paper is exactly parallel to the horizontal rulings. The fitting should be done with nicety. The ordinate to the volume curves shows the amount of materials that can be moved without hauling any of it any greater distance than that marked off on the piece of paper.

Another distance may be marked off on a piece of paper and another limit found. The difference in the amounts of materials found for the two cases will be the amount corresponding to a haul whose length is greater than the lesser distance and does not exceed the greater.

A certain price per cubic yard may be paid for all materials moved not to exceed 300 feet, and another price paid when the haul is from 300 feet to 1,000 feet, while all that is hauled over 1,000 feet may be paid for at an additional price for each cubic yard hauled 100 feet. By means of the haul curve take off the number of cubic yards hauled not to exceed 300 feet; the number hauled not to exceed 1,000 feet, and the corresponding amounts for 1,100 feet, 1,200 feet, 1,300 feet, and so on. The number of cubic yards within the 300 feet limit shows the amount of the item at the first price. The difference between the number of cubic yards within the 1,000 feet limit and what is within the 300 feet limits is the item to be paid for at the second price fixed. The corresponding difference between the number of cubic yards within the 1,000 feet limit and the 1,100 feet limit show the number of cubic yards hauled an extra distance of 100 feet. The corresponding difference for the 1,100 and 1,200 feet limits show the amounts to which an extra haul of 200 feet applies. Similarly the limits 1,200 and 1,300 feet will give the amount to which an extra haul of 300 feet applies. And so on. The sum of the various items will make up the cost.

By the haul curve these various things can be done without finding the centers of gravity at all. Should the actual work be desired an expression for it can be found for any case by multiplying the number of cubic yards to be moved between any limits by the distance between the centers of gravity of the mass moved and the mass it will make. As only the center line positions of these centers of gravity are needed, these positions may be readily found from the haul curve. For any mass between the grade point, G, and a designated cross section, bisect the ordinate to the volume curve at that cross section. Find where a horizontal line through the point of bisection will intersect the volume curve. Here will be the center line position of the center of gravity of the mass between G and the designated cross section. If the same be

done on the other side of G for an equal mass, the distance between the two centers of gravity will be the distance to multiply the number of cubic yards to be moved by in order to get an expression for the work involved.

The center of gravity of any mass between any two ordinates to the volume curve is where the ordinate to the volume curve equals the half sum of these ordinates.

It is not necessary to determine the location of the centers of gravity of the separate prismoids composing these masses. If the volume curves be platted with sufficiently large, and suitably proportioned scales, such matters as are above referred to, may be determined with precision enough for all ordinary cases of moving earth. When the case requires the utmost precision the mthods by calculations, which were previously outlined, can be used, and results obtained that will be as trustworthy as the field notes of the surveys will give.

Perhaps a few general statements relating to volume curves and haul curves may be of service.

55. **Volume Curves.** For any given prismoid the general equation of the volume between A_0, one of the ends of the given prismoid and a section parallel to A_0 at a distance x_1 therefrom is

47. $(V_1)_{x_1} = \frac{1}{3} m_1 x_1^3 + \frac{1}{2} n_1 x_1^2 \times A_0 x_1$

This applies to the prismoid between cross sections A_0 and A_1. For the prismoid between A_1 and A_2 this becomes

48. $(V_2)_{x_2} = \frac{1}{3} m_2 x_2^3 + \frac{1}{2} n_2 x_2^2 + A_1 x_2$

For the prismoid between any two successive cross sections as A_{v-1} and A_v

49. $(V_v)_{x_v} = \frac{1}{3} m_v x_v^3 + \frac{1}{2} n_v x_v^2 + A_{v-1} x_v$

m and n are constants for any given prismoid and are not especially troublesome to compute.

$$50. \quad m = 2\frac{1}{L_1^2}(A_{v-1} - M_v + A_v - M_v)$$

$$= \frac{1}{L_1^2}(A_{v-1} + A_v - K_{v-1} - K_v)$$

$$51. \quad n = -\frac{1}{L_v}\Big[3\,(A_{v-1} - M_v) + A_v - M_v\Big]$$

$$= -\frac{1}{L_v}(2A_{v-1} - K_{v-1} - K_v)$$

Equations 47, 48, and 49, are cubics.

If a curve was platted on cross ruled paper so that ordinates to it were equal to $(V)_x$ for abscissas equal to x, it would be a volume curve. This curve would cut the axis of x at the grade point. Therefore in forming this curve the proper origin is at G, the center line grade point where a fill or cut begins, for which the curve is being laid down. The values of x will then be the distance from G along the center line, and the corresponding ordinates will be values of $(V)_x$. For the prismoid from G to the first cross section the equation for $(V)_x$ can be written and values of $(V)_x$ for any values of x within the length of this prismoid found and platted. These platted points being connected would give the volume curve for that prismoid. When the end of this first prismoid was reached where $x_1 = L_1$ and $(V)_x = V_1$ and these values were platted, a point would be reached that would become the origin for the next piece of the volume curve covering the second prismoid, represented by

$$52. \quad (V_2)_{x_2} = \tfrac{1}{3}m_2\,x_2^3 + \tfrac{1}{2}n_2\,x_2^2 + A_1\,x_2$$

The values of m_2 and n_2 could be calculated, this equation given numerical coefficients and solved. These values of $(V_2)_{x_2}$ and x_2 would enable that part of the volume curve for the second prismoid to be platted. The last point on this piece becomes the origin for platting the piece for the third prismoid from data obtained from an equation for that prismoid. Thence each piece may be platted in turn from the point at the end of the last piece as an origin. This would give a theoretically perfect volume curve made up of pieces of different cubics each corresponding to the prismoid from

which its equation was derived. An ordinate to this curve from the first axis of x at any place would equal the total volume included between G and that place. Consequently that place on this curve where the ordinate equals one-half the ordinate at the end of the curve corresponds to the center of gravity of all the prismoids for which the curve is platted. Likewise that place on this curve where the ordinate equals one-half the sum of two ordinates each way from it, corresponds to the center of gravity of those prismoids coming between those ordinates whose half sum is taken. By these means the center of gravity may be located for any prismoids in any part of the work.

For the pieces of the different cubics constituting the theoretical volume curve it is customary to substitute their chords. This is done by platting, at the location of each station and plus where a cross section was taken, an ordinate equal to the total volume of all the prismoids between G and that station or plus. The points thus located are connected by straight lines forming the chords of the separate pieces of the cubics in the theoretical curve. This broken line is used as the volume curve. Centers of gravity are taken to be where this broken line gives them instead of using the theoretical curve. Ordinates to this broken line are used as if they were ordinates to the theoretical curve. The ordinates to this broken line at the stations and plusses where the cross sections were taken are correct. The approximation is between these places. Should something more exact be required it might involve no more than the piece of the theoretical curve in the immediate vicinity of the point sought. This might not prove to be a very troublesome matter.

56. **Haul Curves.** Two volume curves from the same grade point, one to the fill side and the other to the cut side, constitute a typical Haul Curve. It may happen that more than one cut or more than one fill is involved in each volume curve. Gaps of omitted work may also be included. Nevertheless the typical form above referred to illustrates the primary and essential ideas of a Haul Curve. Various uses are made of the Haul Curve. One characteristic use is that the points of

each volume curve which are on the same horizontal line show the limits of haul. No earth between these limits need be hauled a greater distance than the length (to scale) of the horizontal line between these points. The ordinate at these points shows the volume within this limit of haul. Another use is to ascertain by suitable methods what part of a fill a certain part of a cut will make. This being done, the centers of gravity of each mass is located by the method above suggested. The horizontal distance apart of these centers of gravity is the average haul. This multiplied by the mass gives the work of making that part of the fill from the designated part of the cut.

57. **Shrinkage.** Sand, gravel, and the common forms of loose soil and earth, taken from an excavation will not make the same number of cubic yards in a fill that the excavation measures. More solid materials, from loose rock to solid rock, will make more cubic yards in a fill than these materials occupy before excavation. This difference is called "Shrinkage." It is commonly taken to be a certain percentage of the materials in place before being moved. For the more common earths this is taken to be about ten per cent. For solid rock the increased space occupied in a fill is taken to be about three-eighths more when in large fragments and one-half more when in small fragments. See Searles' Field Engineering pp. 206 and 207. In ordinary earth it is customary to set the stakes and poles to mark the depth of the fill so their tops will be above the ground a distance equal to the fill at the place where they are set, plus ten per cent. This will make an addition to the mass between two cross sections that must be taken into account in various calculations, such as the volume, center of gravity, work, haul, and the balancing of volumes.

Calculations with respect to this added depth of a fill on a formation may be made by means of the following equations:

Let C_0 = the depth of fill at the center stake of any section.

Let C_1 = the depth of fill at the center stake of the succeeding cross section.

Let D_0 = the entire distance between the side stakes of the cross section where C_0 is.

Let D_1 = the entire distance between the side stakes of the cross section where C_1 is.

Let p = the percent of added depth at the center stakes. This will be plus for earth and minus for rock.

Let the rest of the symbols follow the notation already given.

The change in area at any cross section will be as follows:

53. At $A_0 = A'_0 = \frac{1}{2} C_0 (D_0 + 2W) p.$

54. At $A_1 = A'_1 = \frac{1}{2} C_1 (D_1 + 2W) p.$

55. At $M_1 = M'_1 = \frac{1}{4} (C_0 + C_1) \frac{1}{2} (D_0 + D_1 + 4W) p.$

56. $4M'_1 = \frac{1}{2} (C_0 + C_1) (D_0 + D_1 + 4W) p.$

The change in volume of the prismoid between A_0 and A_1 will be

57. $$V'_1 = \frac{1}{6} L_1 \left[\begin{array}{l} \frac{1}{2} C_0 (2D_0 + D_1 + 6W) \\ + \frac{1}{2} C_1 (2D_1 + D_0 + 6W) \end{array} \right] p.$$

The distance from A_0 to the center of gravity of the changed prismoid will be

58. $$x_g = \frac{1}{2} L_1 - \frac{1}{2} L_1 \frac{A_0 + A'_0 - A_1 - A'_1}{A_0 + A'_0 + A_1 + A'_1 + 4M_1 + 4M'_1}$$

59. $$= \frac{1}{2} L_1 - \frac{1}{12} L_1^2 \frac{A_0 + A'_0 - A_1 - A'_1}{V_1 + V'_1}.$$

The work of moving the changed prismoid from its center of gravity to its mid section will be

60. $- \frac{1}{12} L_1^2 (A_0 + A'_0 - A_1 - A'_1).$

From these equations it may be perceived that the same principles, and processes, are to be followed as apply before any change is made. Problems, and graphical constructions, with respect to these changed volumes relating to center of

gravity, work, haul, and the balancing of volumes or items of work, can go on in just the same manner, and by the same processes, as are heretofore outlined.

Another method may be followed in finding how much of a fill a certain portion of a cut will make. Balance off the volumes from the cut against the unchanged volumes of the fill till the termination of the fill of equal volume to the prismoids of the cut is found. Cut off from the outer end of the fill any assumed percentage of its total volume (for shrinkage) by means of a cross section whose location must be found by trial, as before directed. This cross section will be the real limit of the fill which the prismoids from the cut will make. In the case of rock work continue adding prismoids to the outer end of the fill first found, as above, till the total volume of the prismoids of the fill exceed the volume of the prismoids of the cut by the assumed percentage of increase for rock work. The location of the cross section at the outer end of this added fill must be found by trial. This cross section will be the real limit of the fill which the prismoids from the cut will make. In such calculations it is really necessary only to ascertain the location of the cross section at the final limit of the fill so that in the case of earth the total volume of the prismoids of the fill shall be less than the total volume of the prismoids of the cut by the required percentage, and in the case of rock, shall exceed the total volume of the prismoids of the cut by the required percentage.

To allow for "shrinkage" in platting the volume curve for a fill it may be sufficient to increase the ordinates used in platting the same, the assumed percentage for earth; or to decrease them the assumed percentage for rock. A sufficient number of calculations by means of equations 53 to 60 might be made (using a graphical construction to determine the position of the center of gravity of a number of prismoids, if thought best) to verify the results given by the volume curve platted in this manner, and to make sure that they were to be trusted within the limits of precision employed.

In allowing for "shrinkage" on haul curves, the fill curve might be platted as above, the cut curve being platted by the actual volumes. Should a fill curve be required to show the

completed, and settled, work it might also be added to the diagram. Results could be verified by calculations, using equations 53 to 60, until it should be shown whether they were to be trusted or not.

57. **References** for volume and haul curves.

Molitor and Beard's Manual for Resident Engineers, section 65.

Earth and Rock Excavation by Chas. Prelini, chapters II and III.

The Mass Curve, by M. L. Enger, p. 70, Technograph No. 20 for 1905-1906, University of Illinois.

Haul and Overhaul, by H. O. Garman, p. 11, Purdue Engineering Review, 1906.

A Comparison of Some of the Methods of Computing Haul, by H. O. Garman, p. 58, Proceedings Indiana Engineering Society, 1906.

58. **Prismoidal Formulas.**—Here are added a few equations that may be useful to engineers.

Volumes.

Two term formula.

61. $V_1 = \frac{1}{4} L_1 (A_0 + S_{2/3}) = \frac{1}{4} L_1 (A_1 + S_{1/3})$

Three term formula.

62. $V_1 = \frac{1}{6} L_1 (A_0 + 4 M_1 + A_1)$, — same as Eq. 15.

Applies to all volumes having parallel plane ends, A_0 and A_1, the area of any section of which, parallel to its ends, is a function of the distance of such section from A_0 not higher than the second order, that is to say, a function of the general form $(X_1)_{x_1} = A_0 + n\, x_1 + m\, x_1^2$.

Four term formula.

63. $V_1 = \frac{1}{8} L_1 (A_0 + 3S_{1/3} + 3S_{2/3} + A_1)$

Applies to all volumes having parallel plane ends, A_0 and

A_1, the area of any section of which, parallel to its ends, is a function of the distance of such section from A_0 not higher than the third order, that is to say, a function of the general form $(X_1)_{x_1} = (A_0 + n\,x_1 + m\,x_1^2 + q\,x_1^3)$.

Center of Gravity.

64. $x_g = \frac{1}{2}L_1 - \frac{1}{2}L_1 \dfrac{A_0 - A_1}{2A_0 + 2A_1 + K_0 + K_1}$.

65. $\dfrac{x_g}{L_1 - x_g} = \dfrac{A_0 + 3\,A_1 + K_0 + K_1}{3\,A_0 + A_1 + K_0 + K_1}$.

Work.

See Eq. 42 to 45.

Moment of Inertia.

Axis at end A_0.

66. $I_0 = \frac{1}{60}L_1^3\,(\,9\,A_1 + 12\,M_1 - A_0)$

67. $\quad = \frac{1}{60}L_1^3\,(\,2\,A_0 + 12\,A_1 + 3\,K_0 + 3\,K_1)$

68. $\quad = L_1^3\,(\frac{9}{10}\dfrac{V_1}{L_1} - \frac{4}{10}M_1 - \frac{1}{6}A_0)$

69. $\quad = L_1^3\,(\frac{3}{10}\dfrac{V_1}{L_1} - \frac{1}{15}A_0 + \frac{1}{10}A_1)$.

Axis at center of gravity.

70. $I_g = I_0 - x_g^2\,V_1$ is easiest.

Radius of Gyration.

Axis at end A_0

71. $R_0^2 = \dfrac{I_0}{V_1}$

72. $\quad = \frac{1}{10}\,L_1^2\,\dfrac{9\,A_1 + 12\,M_1 - A_0}{A_0 + 4\,M_1 + A_1}$

73. $$= \tfrac{1}{10} L^2_1 \frac{2 A_0 + 12 A_1 + 3 K_0 + 3 K_1}{2 A_0 + 2 A_1 + K_0 + K_1}$$

Axis at center of gravity.

74. $R^2_g = R^2_0 - x^2_g$ is easiest, or use "Note" below.

Center of Percussion.

Axis at end A_0.
Distance from A_0.

75. $$P_0 = \frac{R^2_0}{x_g}$$

76. $$= \tfrac{1}{10} L_1 \frac{9A_1 + 12M_1 - A_0}{A_1 + 2M_1}$$

77. $$= \tfrac{2}{10} L_1 \frac{2A_0 + 12A_1 + 3K_0 + 3K_1}{3A_0 + A_1 + K_0 + K_1}.$$

Axis at center of gravity.

Distance from

78. $P_g = P_0 - x_g$ is easiest.

"Note." To find R_g^2, use the principle that

79. $(P_0 - x_g)\, x_g = R_g^2$, having found P_0 and x_g by previous equations.

While 64 to 79 are special equations they are sufficient for much of the work of an engineer. They are derived from 16, or 62.

Apply to Areas.

All these formulas apply to areas, between parallel ends. A_0, A_1, and M_1, then represent breadths, instead of areas.

In all these equations, either in their applications to volumes or areas, A_0, A_1 or M_1, may equal zero. They apply to solids of revolution where the generatrix is a straight line or a